An understanding of the Internal Revenue Code time value of money sections requires a working knowledge of financial calculations. Inexpensive calculators have alleviated the need for lawyers to understand the actual formulas; however, because the calculators and their respective manuals are often complicated, lawyers lacking an accounting or finance background may shy away from this important area of tax law.

This chapter serves two purposes: First, it provides a basic explanation - with lawyers as the intended audience - of the use and application of a typical financial calculator, the Hewlett Packard 10B. Second, it discusses the legal system’s use of financial terminology.

I. USE OF A CALCULATOR

My first advice is “Read the manual.” Most financial calculator manuals explain the various types of calculations and provide understandable examples. This chapter is not intended to pre-empt or replace that advice. Nevertheless, having helped a great many law students and lawyers use a variety of calculators, I have learned that many well-educated people have trouble with manuals.

A. TYPES OF CALCULATIONS

While financial calculators can compute many things, six types of calculations are fundamental:

1. Present Value of a Sum
2. Future Value of a Sum
3. Present Value of an Annuity
4. Future Value of an Annuity
5. Sinking Fund
6. Amortization
Each of these calculations is relevant to one or more Internal Revenue Code provisions. Each also relies on the same basic formula, involving six factors, with the typical key label:

1. The present value (PV)
2. The future value (FV)
3. The interest rate per year (I/YR)
4. The number of periods per year (P/YR)
5. The amount of each payment (PMT)
6. The number of periods (N)

In the typical example, five of the six factors will be known. The calculator can then easily solve for the sixth. In addition, annuity calculations require a mode setting, indicating whether payments occur at the beginning or the end of a period.

**B. COMMON DIFFICULTIES**

Unless the calculator is defective, which is unlikely, it will produce the correct answer if given the correct information. Nevertheless, many users, at one time or another, exclaim “This thing doesn’t work!” Usually, they have violated one of the following rules:

1. **First, clear the machine.** It knows only what you tell it and it does not forget until you tell it to forget, typically even if you turn off the machine. Thus, be certain to clear all functions and memory when beginning a new calculation. All calculators have a clear key, usually denominated with a C or the word clear. In addition, many calculators have a function key by which merely the last information entered can be cleared, and a different function key by which all information can be cleared.

   For example, the HP 10B calculator has three levels for the clear function.

   1. The C key will clear the entire displayed number; however, it leaves the memory intact. For example, if you input 50+ 20+ 30 but intended 50+ 20+ 40 press this key, erasing the 30 but leaving the 50 + 20 in memory. You can then press 40 and =. The display will then read 110.

   2. The ← key will clear single digits, one at a time. For example, if you input 523 but intend 524, you may use this key to erase the 4. In contrast, the C will clear the entire number 523.
3. The key [INPUT] when pressed in the “shifted mode” will clear the entire memory, as well as the displayed number. To perform this function, first press the orange key [CLEAR ALL]. This key shifts the function to CLEAR ALL rather than INPUT. Before working a new problem, you will usually want to press this key. This function, however, does not re-set the number of periods per year. If you change this setting, it will remain - even if you turn off the calculator - until you manually change it again. Also, this clear all function does not change the mode. Thus if you re-set the mode from end to begin, or vice versa, it will remain - even if you turn off the calculator - until you re-set it manually.

2. Set the cash flows with the proper sign. Many, but not all, calculators require that cash flows be directional. This means that one set must be positive and the other negative. For example, in such machines the present value may be expressed as a positive number - a deposit - while the future value will be expressed as a negative number - a withdrawal. Or, the opposite may be true; however, the present and future values cannot both be positive or negative at the same time. On the other hand, some calculators eliminate this feature. Hence, be sure to read your own manual.

The HP 10B calculator requires that cash flows be entered with opposite signs. Failure to do so will prompt the display no Solution. For example, suppose you want to compute the annual interest rate inherent in a problem with a present value of 500, a future value of 1000 and 10 years. The correct answer is 7.177346254. To achieve this, either the 500 or the 1000 must be expressed as a negative number while the other must be positive.

A negative number may be entered in two ways. For example, to input the number (1000), first enter the positive number 1000, then press the “plus/minus” key:

1000 +/-

This will change the sign from positive to negative or from negative to positive. In the alternative, press the minus sign, the number, and then the equal sign, as follows:

- 1000 =

3. Set the mode correctly. Calculations involving annuities, including sinking funds and amortizations, require a “mode,” either begin or end. Use begin mode if payments (or deposits or withdrawals) occur at the beginning of each period. Use end mode if payments occur at the end of each period. Typically, a sinking fund uses the begin mode, while an amortization - such as the repayment of a loan - uses the end mode. “Future Value of a Sum” and “Present Value of a Sum” calculations are not affected by the mode setting.

An HP 10B calculator has a key [beg/end] which serves both as the “0” and as the mode setter. First, press the key [beg/end] to operate the mode function. Most calculators are pre-set at the factory in end mode. Pressing these two keys will change it to begin mode, which the display will
note. To revert to end mode, press the two keys again. The display will no longer indicate the mode. If you change the setting to begin mode, it will remain, even if you turn off the calculator or utilize the clear all function. To revert to end mode, you must do so manually.

4. Set the interest rate to compound for each payment period. In other words, the compounding period and the payment period must be the same. Thus, if the facts provide for annual payments, the interest rate must be stated as an annual rate. If, instead, the facts provide for semi-annual payments, then the interest rate must be stated as a semi-annual rate. Likewise, monthly payments call for a monthly interest rate.

If the stated interest compounding period and the payment period are not the same, you can easily convert the interest rate to an equivalent one using a compounding period identical with that of the payments. Some calculators do this automatically with a feature labeled IConv. Others require the user to make the computations, which are not difficult.

For example an “annual percentage interest rate”\(^2\) of 10% with semi-annual payments is not the equivalent of a semi-annual rate of 5%. Instead, it is the equivalent of 4.8808848170% semi-annually. In other words, 4.88% interest, compounded semi-annually is the same as 10.00% interest, compounded annually. In contrast, of course, two $500 payments - one each six months - is not the equivalent of a single $1000 payment. Thus, if on the facts given, the interest compounding period and the payment period are not co-extensive, you must convert the interest rate to the payment period, rather than convert the payment to the stated compounding period.

5. Set the number of periods per year correctly. Most calculators are factory preset for twelve periods per year. All, however, can easily be re-set, when indicated for a particular problem. For example, in a problem involving a single annual payment, the payments per period function must be set at 1.

To change the factory setting on an HP 10B, press the desired number - 1 - then press † and then \( \text{P/yr} \). This puts the key into the \( \text{P/YR} \) function. You can check the setting by pressing † and then \( \text{clear all} \). The display will indicate the number of payments per year. This will remain as the setting - even if you turn off the calculator or utilize the clear all function - until you manually re-set it.

Changing the payments per year setting is not always necessary. Some people find it easier to set the \( \text{P/Yr} \) function to one, meaning one payment per period. Then, in entering an interest rate they always enter a periodic rate. For example, if the problem calls for ten years of monthly payments at 12% nominal annual interest, the calculator is indifferent to whether it is told 12 P/Yr at 12% interest or 1 P/Yr at 1% interest. In either case, the N setting must be 120 to indicate the correct

---

\(^2\) I use this term carefully. See infra at page _____ the definitions of various terms.
number of payments. To prove this, compute the present value of an annuity in arrears of 1000 per month for ten years at 12% nominal annual interest.

With an HP 10B, press the following keys:

```
[clear all]
1000 [PMT]
1 [I/YR]
120 [N]
0 [FV]
PV
```

The display will read -69,700.5220314.

Or, using the same calculator, press the following keys:

```
[clear all]
1000 [PMT]
12 [I/YR]
12 [P/YR]
12 [P/M]
0 [FV]
120 [N]
PV
```

The display will read -69,700.5220314.
The display will again read - 69,700.5220314. In this second method, the calculator divided the nominal interest rate by the number of payments per year to achieve the correct periodic interest rate. In the previous method, the calculator was told the periodic interest rate. As a practical matter, some people find it easier to remember always to enter to correct periodic rate than to remember always to enter the correct number of payments per year. They thus set the calculator to one payment per year and leave it at that setting for all calculations. Whichever method works best for the individual user is the one he or she should use.

6. Set the display to the correct number of decimal places. Although most calculations involve dollars and cents and thus two places after the decimal, large numbers and long periods of time can be significantly affected by rounding. While the calculator internally uses twelve places after the decimal for calculations, it displays only the number pursuant to its setting. Because some calculations involve the user writing down or otherwise re-using a computed number, it may be helpful to have the display read the full nine spots after the decimal. As with most calculators, the factory setting of an HP 10B is for two places after the decimal. To change this to nine - the maximum permanent display - press the following keys:

\[
\begin{align*}
\text{disp} & \quad 9 \\
\end{align*}
\]

The display will then show nine places after the decimal. To change this to any other number from one to eight, re-enter the key strokes, using the desired number of places. Or, to display twelve numbers (with no decimal place) press

\[
\begin{align*}
\text{disp} & \quad = \\
\end{align*}
\]

and hold down the equal sign. The display will temporarily show twelve digits.

C. CALCULATIONS

1. Future Value of a Sum

This calculation computes the future amount of a current deposit. For example, $1,000 deposited today, earning 10% interest compounded annually, will increase to $1,100 in one year. To calculate this, input the five known factors into the calculator and solve for the unknown sixth factor, the future value. First, set the Present Value (PV) as 1,000.00. Set the Interest (I/YR) rate per period as 10. Set the Number of Periods per year (P/Yr) as 1. Set the Payment (PMT) amount as 0. Set the Number of Periods (N) as 1. Finally, solve for the Future Value (FV) by pressing the FV key. The answer will appear as (1,100.00), the negative indicating a withdrawal.

With an HP 10B, press the following keys (other than the first and last functions - clearing and solving - the other you input the data is unimportant):
In the preceding example, the unknown factor was the future value. If, instead, you knew the present value, the future value, and the number of periods, payments, and payments per period, you could also use the above method to solve, instead, for the interest rate per period. Set the \( PV \) as 1,000.00, the \( FV \) as -1,100.00, the \( N \) as 1, the \( P/YR \) as 1, the \( PMT \) as 0, and press the \( I/YR \) key to solve for the unknown annual interest rate.

With the HP 10B calculator, press the following keys:
b. Converting an interest rate

Recall that one of the cardinal rules of financial calculations is that the interest rate period and the payment period must be the same. For example, if payments - either of principal or interest - occur semi-annually, then the interest rate must also be stated as a semi-annual rate. Or, the payments occur quarterly, the rate must also be expressed as a quarterly rate. This is not really much of a problem, however, because the typical calculator can easily convert an interest rate to an equivalent rate for another period. Many calculators do this automatically. You can also perform this function manually.

As explained above, 4.8808848% interest, paid semi-annually is the equivalent of 10% interest, paid annually. This is true because the interest paid during the first six month period will itself earn interest of 4.88% during the second six month period. To prove this, set PV as 1,000.00, I/YR as 4.8808848, P/Y as 1, PMT as 0, and N as 2. Press FV. The answer is (1,099.99999964), which is the equivalent of (1,100.00). Hence, $1,000 earning 10% interest annually produces the same result as $1,000 earnings 4.88% semi-annually. This example also demonstrates that the I/YR key can function as an interest per period key and the P/Y key can be a number of periods key, rather than number of periods per year.

Similarly, to convert the initially known effective annual interest rate of 10% to an equivalent semi-annual periodic rate set PV as 1,000.00, P/Y as 1, PMT as 0, N as 2, and FV as (1,100.00). Then, press I/YR to solve for the periodic interest rate of 4.8808848. Multiplying the result by two will produce the nominal annual interest rate, which is compounded semi-annually, of 9.761769634.

An alternative method utilizes the calculator’s built-in conversion function. Enter 10.00 as the effective interest rate using the EFF% function, then solve for the nominal interest rate, using the NOM% key. Dividing the result by 2 would produce the semi-annual rate.

Using an HP 10B calculator, press the following keys to produce the above result:
The display will read 9.761769634, which is the nominal annual interest rate, compounded semi-annually, equivalent to 10.00 effective interest rate, or in the above examples, the annual percentage yield.

2. Present Value of a Sum

This calculation computes the present value of a future amount. For example, $1,100 in one year, discounted at 10% interest compounded annually has a present value of $1,000.00 today. To calculate this, input the five known factors into the calculator and solve for the unknown sixth factor, the present value. First, set the Future Value (FV) as 1,100.00. Set the Interest (I/YR) rate per period as 10. Set the Number of Periods per year (P/Yr) as 1. Set the Payment (PMT) amount as 0. Set the Number of Periods (N) as 1. Finally, solve for the Present Value (PV) by pressing the PV key. The answer will appear as (1,000.00), the negative indicating a current deposit.

With an HP 10B, press the following keys:

```
1 clear all

1100 FV

10 I/YR

1 P/YR

0 PM

1 N

PV
```

The display will read -1,000.000000.
3. Present Value of an Annuity

a. End Mode - An Annuity in Arrears

This calculation computes the present value of a series of equal payments made at the end of regular intervals, earning a constant interest rate. For example, $1,000 deposited at the end of each year for ten years, earning 10% interest compounded annually, has a present value today of $6,144.57. To calculate this, input the five known factors into the calculator and solve for the unknown sixth factor, the present value. First, set the Payment (PMT) as 1,000.00. Set the Interest (I/YR) rate per period as 10. Set the Number of Periods per year (P/Yr) as 1. Set the Number of Periods (N) as 10. Set the Future Value (FV) as 0. Solve for the Present Value (PV) by pressing the PV key. The answer will appear as (-6,144.56710570), the negative indicating a required deposit necessary to generate the level annuity. The Future Value is zero because at the end of ten years, no money would remain in the account.

With an HP 10B, press the following keys:

```
clear all
1000 PMT
10 I/YR
1 P/YR
1 PM
0 FV
10 N
PV
```

The display will read - 6,144.56710570.

b. Begin Mode - An Annuity Due

This calculation computes the present value of a series of equal payments made at the beginnings of regular intervals, earning a constant interest rate. For example, $1,000 deposited at the
beginning of each year for ten years, earning 10% interest compounded annually, has a present value
today of $6,759.02. The amount exceeds that of the above calculation because the first payment here
is made today, whereas in the End Mode (Annuity in Arrears), the first payment is not made until one
year from now.

To calculate this, input the five known factors into the calculator and solve for the unknown
sixth factor, the present value. First, set the calculator in Begin Mode. Then, set the Payment
(PMT) as 1,000.00. Set the Interest (I/YR) rate per period as 10. Set the Number of Periods per
year (P/Yr) as 1. Set the Number of Periods (N) as 10. Set the Future Value (FV) as 0. Solve for
the Present Value (PV) by pressing the PV key. The answer will appear as (6,144.56710570), the
negative indicating a required deposit necessary to generate the level annuity. The Future Value is
zero because at the end of ten years, no money would remain in the account.

With an HP 10B, press the following keys:

```
clear all
Input
BEG/END
0
1000
PMT
10
I/YR
1
P/YR
0
FV
10
N
PV
```

The display will read   - 6,759.02381627.
4. Future Value of an Annuity

a. End Mode - An Annuity in Arrears

This calculation computes the future value of a series of equal payments made at the end of regular intervals, earning a constant interest rate. For example, $1,000 deposited at the end of each year for ten years, earning 10% interest compounded annually, has a future value in ten years of $15,937.42. To calculate this, input the five known factors into the calculator and solve for the unknown sixth factor, the present value. First, set the Payment (PMT) as 1,000.00. Set the Interest (I/YR) rate per period as 10. Set the Number of Periods per year (P/Yr) as 1. Set the Number of Periods (N) as 10. Set the Present Value (PV) as 0. Solve for the Future Value (FV) by pressing the PV key. The answer will appear as (15,937.4246010), the negative indicating withdrawal possible after the ten deposits. The Present Value is zero because at the beginning, no money has yet been deposited.

With an HP 10B, press the following keys:

\[
\text{clear all} \\
\text{1000} \rightarrow \text{PMT} \\
\text{10} \rightarrow \text{I/YR} \\
\text{1} \rightarrow \text{P/YR} \\
\text{0} \rightarrow \text{PV} \\
\text{10} \rightarrow \text{N} \\
\rightarrow \text{FV}
\]

The display will read \(-15,937.4246010\).

b. Begin Mode - An Annuity Due

This calculation computes the future value of a series of equal payments made at the beginnings of regular intervals, earning a constant interest rate. For example, $1,000 deposited at the beginning of each year for ten years, earning 10% interest compounded annually, has a future value
in ten years of $17,531.17. The amount exceeds that of the above calculation because the first payment here is made today and thus earns interest beginning today, whereas in the End Mode (Annuity in Arrears), the first payment is not made until one year from now.

To calculate this, input the five known factors into the calculator and solve for the unknown sixth factor, the present value. First, set the calculator in Begin Mode. Then, set the Payment (PMT) as 1,000.00. Set the Interest (I/YR) rate per period as 10. Set the Number of Periods per year (P/YR) as 1. Set the Number of Periods (N) as 10. Set the Present Value (PV) as 0. Solve for the Future Value (FV) by pressing the FV key. The answer will appear as (17,531.1670611), the negative indicating a required deposit necessary to generate the level annuity. The Present Value is zero because at the beginning, the account contains no money: the purpose of the computation is to determine the necessary deposits.

With an HP 10B, press the following keys:

```
clear all
BEG/END
1000 PMT
10 I/YR
1 P/M
0 PV
10 N
FV
```

The display will read -17,531.1670611.

5. Amortization

This calculation solves for the amount of the regular payment needed, at a stated interest rate and period, to pay off a present value. This is the opposite of the above calculation involving the
Present Value of an Annuity. Amortization schedules typically involve the end mode because loan payments generally occur at the end of each period, rather than at the beginning.

For example, if you were to borrow $6,144.57 today and agreed to make ten equal annual payments at an annual interest rate of ten percent, each payment would need to be $1,000. To calculate this, set the calculator in end mode, set the Present Value (PV) as 6,144.57, the Future Value (FV) as 0, the Interest Rate per Year (i/yr) as 10, the Number of Payments Per Year (p/yr) as 1, and the Number of Payments (n) as 10. Then solve for the amount of the Payment (pmt). The displayed answer will be -1000.00, the negative indicating the payment.

With an HP 10B, press the following keys:

1. Clear all
2. Input
3. 10
4. I/YR
5. 1
6. P/YR
7. 1
8. PM
9. 6,144.57
10. PV
11. 10
12. N
13. 0
14. FV
15. pmt

The display will read -1,000.00047103. The extra digits (.00047103) round to zero. Thus the necessary payment is $1,000, which includes both the interest and principal.

With many calculators, you may obtain a schedule breaking down each of the payments into the component parts of interest and principal, as well as the balance due. Some calculators, such as the HP 200LX perform this function easily, displaying the components neatly on the screen. Other calculators, however, such as the HP 10B use a smaller screen and a more awkward way of displaying the results.

To perform the amortization schedule function, using the HP 10B calculator, first compute the amount of the payments, as explained above. Then press the following keys:

1. Amort
2. FV
This shifts the calculator to use the key’s Amortization (amort) function, rather than the Future Value (FV) function. The display will read Per 1-1. This indicates Period 1.

Next, press the key indicating the equal sign, holding it down temporarily. While it is held down, the display will read Int, indicating the interest included in the payment for period 1. Then, release the key. The display will then read -614.457, which is the amount of the period 1 interest.

Next, press the key again, also holding it down temporarily. While it is held down, the display will read Prin, indicating the principal included in the payment for period 1. Then, release the key. The display will then read -385.543471030, which is the amount of the period 1 principal.

Next, press the key again, also holding it down temporarily. While it is held down, the display will read bal, indicating the balance due on the loan at the end of the first period. Then, release the key. The display will then read 5,759.02652897, which is the balance due at the end of period 1, after the initial payment of $1,000.00. Naturally, this amount rounds to $5,759.03. If you then press the equal sign key once again, the display will revert to Per 1-1. You could then repeat the above three steps, once again showing the interest, principal, and balance for period one.

Next, once again press . The display will read Per 2-2. This indicates period 2. Repeating the above three steps will cause the calculator to display successively the interest principal and remaining balance for the second period. This process can then be repeated for periods three through ten, at which time the balance will be zero.

6. Sinking Fund

This calculation solves for the amount of the regular deposit needed, at a stated interest rate and period, to accumulate a future value. This is the opposite of the above calculation involving the Future Value of an Annuity. Sinking Fund schedules often involve the begin mode because savings plan deposits often begin at the inception of the plan, which would be the beginning of the first period. The end mode calculation, however, may also be used.

a. Begin Mode

For example, if you wanted to accumulate $17,531.17 in ten years and were willing to make ten equal annual deposits, beginning today, at an annual interest rate of ten percent, each deposit would need to be $1,000. To calculate this, set the calculator in begin mode, set the Present Value (PV) as 0, the Future Value (FV) as 17,531.17, the Interest Rate per Year (i/yr) as 10, the Number of Payments Per Year (p/yr) as 1, and the Number of Payments (n) as 10. Then solve for the amount of the Payment (pmt). The displayed answer will be -1000.00, the negative indicating the deposit.
With an HP 10B, press the following keys:

```
| Clear all                  |
| Input                      |
| Beg/end                    |
| 0                          |
| 10                         |
| I/YR                       |
| 1                          |
| P/YR                       |
| 1                          |
| P/M                       |
| 0                          |
| PV                         |
| 10                         |
| N                          |
| 17,531.17                  |
| FV                         |
| pmt                        |
```

The display will read \(-1,000.00016764\). The extra digits (.00016764) round to zero. Thus the ten necessary deposits are each $1,000, which, with accumulated interest, will equal $17,531.17 in ten years.

b. End Mode

If you wanted to accumulate $15,937.43 in ten years and were willing to make ten equal annual deposits, beginning one year from today, at an annual interest rate of ten percent, each deposit would again need to be $1,000. Notice that by shifting to end mode - thus delaying each payment one year - the Future Value accumulated is substantially less than in the preceding problem.

To calculate this, set the calculator in end mode, set the Present Value (PV) as 0, the Future Value (FV) as 15,937.43, the Interest Rate per Year (i/yr) as 10, the Number of Payments Per Year (p/yr) as 1, and the Number of Payments (n) as 10. Then solve for the amount of the Payment (pmt). The displayed answer will be -1000.00, the negative indicating the deposit.

In the alternative, you might have worked the preceding problem, computing the sinking fund
amount necessary to accumulate $17,531.17 in ten years, using the begin mode. Perhaps, you then
would want to compute the amount you would instead accumulate, with the same payment amounts
and interest rate, but using the end mode (delaying each payment for one year). To compute this, you
would compute the Future Value of an Annuity in Arrears of $1,000 per year for ten years at ten
percent annual interest. This example is worked above in the section on Annuities. This illustrates
that the Sinking Fund and Future Value of an Annuity calculations are merely the reverse of each
other - one knows the Future Value and solves for the payment, while the other knows the payment
but solves for the Future Value.

To work the Sinking Fund problem with an HP 10B, press the following keys:

```
Abbreviation    Description
---             -----------
clear all       clear all
10              I/YR
1               P/YR
0               PM
10              N
15,937.43       FV
pmt
```

The display will read - 1,000.00033876. The extra digits (.00033876) round to zero.
Thus the ten necessary deposits are each $1,000, which, with accumulated interest, will equal
$15937,43 in ten years.

**PROBLEMS**

1. Future Value.
   a. Compute the future value of $1,000.00 in 10 years at 6% nominal annual interest compounded annually.
   b. Compute the same amount, but compound the rate semi-annually.
   c. Compute the same amount, but compound the rate monthly.
d. Compute the same amount, but compound the rate daily.

2. Present Value.

a. Compute the present value of $100,000.00 to be received 18 years from today. Use a nominal annual interest rate of 6% and compound it annually.

b. Compute the same amount, but compound the rate semi-annually.

c. Compute the same amount, but compound the rate monthly.

d. Compute the same amount, but compound the rate daily.

3. Future Value of an Annuity.

a. Compute the future value of $1,000.00 to be paid annually for ten years, the first payment due at the beginning of the period (an annuity due). Use a nominal annual interest rate of 6% and compound it annually.

b. Compute the future value of $1,000.00 to be paid annually for ten years, the first payment due at the end of the period (an annuity in arrears). Use a nominal annual interest rate of 6% and compound it annually.

c. Compute the same amounts as above - both the annuity due and annuity in arrears - but compound the rate semi-annually. In computing these amounts, note that you cannot convert the annuity from ten $1,000.00 payments to twenty $500.00 payments, as that would produce an incorrect answer. You must, instead, convert the interest rate from a nominal annual rate to an annual percentage rate. This is a common problem. In real life, the I.R.S. converts the interest rates for you and announces the conversions monthly. However, I want you to be able to do it for yourself so that you can understand the process.

4. Present Value of an Annuity.

a. Compute the present value of $1,000.00 to be paid annually for ten years, the first payment due at the beginning of the period (an annuity due). Use a nominal annual interest rate of 6% and compound it annually.

b. Compute the present value of $1,000.00 to be paid annually for ten years, the first payment due at the end of the period (an annuity in arrears). Use a nominal annual interest rate of 6% and compound it annually.

c. Compute the same amounts as above - both the annuity due and annuity in arrears - but compound the rate semi-annually.

5. Sinking Fund.

a. Suppose you need $100,000.00 18 years from now for your child's education. Assume a nominal annual interest rate of 6% to be compounded monthly. How much must you deposit each month to attain your goal. Assume an annuity due. Ignore any income tax consequences or inflation.

b. Suppose you have no idea what you will need for your child in 18 years for his education. However, you estimate that if he were to reach 18 now, you would feel comfortable having $50,000.00 in savings for him (you figure he can get a job for any additional needs). You do not know what the inflation rate will be for education costs; however, you estimate that it will average an amount close to the inflation rate for the general economy. You also do not know future tax rates; however, you assume they will be 39% for the first 14 years and 15% for the next 4 years. You know that you must make quarterly estimated tax payments on any earnings; however, for ease of computation, you may assume
that such payments are made monthly. How much must you deposit each month to attain your goal. Assume an annuity due. [This problem is quite beyond our course; however, I include it in case you want to see a real life problem].

6. Amortization Schedule.

a. You borrowed $150,000.00 from your grandmother to purchase a new home. She wants to earn an annual percentage rate of 7% on her money. How much will your monthly payments be on a thirty year loan?

b. Compute the same amount, however, do it for a fifteen year loan.

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**ADDITIONAL PROBLEMS**

I will not spend class time on the following problems. They are, however, representative of real life situations. Some of these you may face. If you are not confident that you can work all of them accurately, let me know.

1. Your grandmother just gave your newborn child $20,000 for her college education. Suppose you can earn 8% nominal interest, tax-deferred, compounded alternatively annually, semi-annually, and monthly. How much will you have when she is 18?

2. Suppose you believe you need $100,000 18 years from now for your child's education. How much would you need to deposit today, tax deferred, to insure that amount? Suppose you can earn alternatively 5%, 8%, and 12% nominal interest compounded alternatively annually and monthly.

3. Suppose your client needs $250,000 to satisfy an obligation four years from now, assuming he loses a lawsuit. He wants to deposit an appropriate amount today to a designated settlement fund under § 468B of the Internal Revenue Code. Assuming he can earn 6% interest tax free compounded annually. How much must he deposit? Assume that he can earn 12% taxable interest (at a 33.33% rate) compounded annually.

4. Suppose you just won the lottery which will pay $1,000,000 per year for 20 years. What is the present value of your winnings, assuming alternative nominal interest rates of 5% and 10% alternatively compounded annually and monthly?

5. Your client was injured. At the time of her injury she was a heavy equipment operator earning $35,000 per year before taxes. Her remaining work expectancy was 40 years. She is not now capable of earning anything. What should she receive for lost wages, assuming a discount rate of 5% compounded annually? What about a rate of 3% compounded annually? Or, 2.5%? Suppose she would have paid $10,000 in income and employment taxes on her gross income. How would that affect the current value?

6. You represent wife in a dissolution action for equitable distribution of property. Husband is entitled to a retirement annuity equal to 75% of his average salary upon retirement. He is now thirty. He has a life expectancy (according to insurance tables) of eighty. He may retire at age sixty-rive and then begin drawing on the retirement benefits. He currently earns $60,000 per year. His retirement benefits are a marital asset under Florida law because the present value of those benefits has been wholly earned during the marriage. Wife would rather receive property equal to the present value of one-half the benefits rather than a QDRO [qualified domestic relations order] pursuant to I.R.C. § 414(p) entitling her to on-half the future benefits when paid. How much should she demand? Do you want more information regarding the retirement plan? What information? Why?

7. You represent a young pitcher. The Atlanta Braves have just drafted him and offered him $3.0 million per year for six years. How much is that worth today? Naturally, you need more information. What information do you need?
Why?

8. You want to borrow $20,000 to purchase a new car. The nominal annual interest rate is 10%. What will be your monthly payments for interest and principal on a five year loan? On a four year loan? On a three year loan?

9. You want to borrow $100,000 to purchase a home. The nominal annual interest rate is 8%. What will be your payments for interest and principal on a fifteen year loan? On a thirty year loan? Suppose instead that the nominal interest rate is alternatively 6%, 8.5%, or 12%.

10. You want to borrow $100,000 to purchase a home. The nominal annual interest rate is 9% compounded monthly. The bank charges three points on the loan, which must be paid separately. What will be your payments for interest and principle on a fifteen year loan? On a thirty year loan? What is the annual percentage rate? How would your answer change if the bank allows you to deduct the points from the amount of the loan?

11. You want to accumulate $1,000,000 by the time you retire at age 65. You are now 25. You assume a nominal annual interest rate of 6% to be compounded monthly. How much will be your deposits if you assume level contributions for the entire forty years?

12. Your client needs to accumulate $50,000 in five years for a balloon payment on a note. He wants to fund it monthly. What would be his deposits, assuming a nominal annual interest rate of 6% to be compounded monthly? 12%?

13. In problem 11, you wanted to accumulate money for retirement; however, you do not know with any confidence what $1,000,000 will be worth in 40 years. Therefore, the problem is not realistic. How could you determine the amount of needed contributions as a percentage of your monthly income? Could you also work this problem using an amortization table? Why?

II. DEFINITIONS

A. Interest Rate

Standing alone, the term interest rate has no useful meaning. Instead, it requires one or more modifiers to indicate the period and frequency of compounding. Four different descriptions of interest are common. Each has its own appropriate use; thus, no description is correct or incorrect: they simply have different meanings

1. Nominal annual interest rate. Also sometimes called the “stated interest rate” or “coupon rate” this is the periodic interest rate times the number of periods per year. Thus an interest rate of one percent per month produces a nominal annual interest rate of twelve percent per year. This number is necessary for calculation involving multiple periods per year because the inverse of the equation is also true: the periodic interest rate equals the nominal rate divided by the number of periods per year. Because the interest compounding period and the payment period must be the same, calculations involving multiple annual payments require the calculation of a periodic rate, which itself requires the use of the nominal rate.

Whenever the interest compounds annually, the nominal annual interest rate will equal the effective interest rate. However, whenever the interest compounds more often than annually, the
nominal annual interest rate will be less than the effective rate. This can result in some persons being misled by the statement of an interest rate. For example, a document may refer to a nominal rate of 10%, while later providing for monthly compounding. The effective interest rate will be 10.471306744%. A reader who does not appreciate the difference between the two rates - and their uses - may mistakenly visualize a lower rate of interest for the transaction than is accurate. In any event, the statement of the 10% nominal rate would be correct albeit easily misunderstood.

Using an HP 10B calculator, convert a 10% nominal annual interest rate, compounded monthly, to the equivalent effective interest rate by pressing the following keys:

- clear all
- Input
- 12 P/yr
- 10 NOM %
- I/YR
- EFF%
- PV

The display will read 10.471306744. Reverse the process to convert the effective rate to the nominal rate.

2. **Periodic Interest Rate.** This is the amount of interest per period. Any calculation involving multiple annual payments requires the use of a periodic rate. Many legal documents will state a periodic rate, as well as the equivalent nominal annual rate and/or the equivalent effective annual rate. For example, a periodic rate of 1% per month is the equivalent of a nominal annual rate of 12% and an effective annual rate of 12.682503013%. The periodic rate is necessary for any calculations. In addition, it is useful if the transactions involves less than a full year.

To calculate the periodic rate, divide the nominal rate by the number of periods per year. When using an HP 10B calculator, two methods are available:

a. **Method One.** Set the periods per year as one. Divide the nominal annual interest rate by the number of periods per year (the real number, not one) and use the answer as the interest rate for the I/Yr function.

b. **Method Two.** Set the periods per year correctly. Do not compute the
periodic rate. Instead input the nominal annual interest rate for the I/Yr function. The calculator will compute the periodic rate.

3. Effective Interest Rate. This term has the same general meaning as the annual percentage yield or the yield to maturity and a similar meaning to the term internal rate of return. The four similar terms, however, have their own uses and are not precisely interchangeable.

a. Deposits. For original deposits, with no withdrawals, each of the four terms will be the same. The effective interest rate will be the annual compounded rate of interest: the actual amount of interest earned for a particular year divided by the amount on deposit at the beginning of the year. Financial institutions generally quote this rate in terms of an annual percentage yield. In other words, it is the periodic rate compounded for the appropriate number of periods for an entire year. This is the most useful number for purposes of comparing one deposit with another.

For example, one financial institution may offer 10% nominal annual interest compounded semi-annually, while another offers 9.9% nominal annual interest compounded quarterly, and a third offers 9.8% compounded daily. A comparison of those three rates is difficult because of the differing compounding periods. Stating each in terms of an effective annual rate eliminates any confusion. The first institution is actually offering 10.25% effective interest. The second is offering 10.273639392% effective interest, slightly more than the first even though it offers a lower nominal rate. The third institution is offering 10.294827794, more still even though it offers the lowest of the three nominal rates.

The nominal annual interest rate for a deposit will always be lower than the effective interest rate. As a result, financial institutions will always quote, in the most prominent language, the effective interest rate or annual percentage yield on a deposit.

Accounts which have occasional withdrawals or additional deposits will have the same effective interest rate and annual percentage yield or yield to maturity; however, they may have a different internal rate of return. Sales of a debt instrument subsequent to issue and prior to maturity - or offers to sell it - may result in a different yield to maturity and internal rate of return, because of the changing present value as a result of market forces.

b. Loans. Discount loans with no payments prior to maturity and no points will have an effective interest rate equal both to the annual percentage yield and the nominal annual rate. They will also have an annual percentage rate equal to the nominal rate. Installment loans and loans with points, however, will have differing effective interest rates, nominal rates, and annual percentage rates.

The effective rate on an installment loan with no points will be the interest rate that would accrue annually if the interest on the loan compounded. In actuality, interest on an installment loan without negative amortization does not compound; instead, the installments pay the interest due plus, usually, a portion of the principal. As a result, no interest is charged on interest. In a sense, the effective
interest rate on such a loan is not representative of reality: while the effective rate is a compounded rate, the actual interest on the loan does not compound.

Nevertheless, the effective rate on such a loan is a useful number. It reflects what would happen if the interest compounded. In reality, the interest does compound, though not specifically with regard to the loan instrument. This is true both from the standpoint of the lender and the borrower. From the lender’s viewpoint, he receives installment payments, including all interest due and some principal. Those amounts do not earn additional interest from this borrower with regard to this loan; however, the lender must do something with the funds. If deposited or loaned elsewhere, they will earn additional interest. If expended, they will free up other funds which can earn interest, or they will reduce the need for borrowing, which will reduce other interest costs. Thus, effectively, the funds earn interest for the entire year (unless the applicable currency is stuffed in a mattress or some other unproductive investment). Stating the uncompounded periodic rate on the particular loan as a compounded effective rate reflects the reality that the funds will earn interest from some source for the entire year.

From the borrower’s viewpoint, he makes installment payments, including all interest due and some principal. As a result, he does not owe additional interest on those funds to that lender with regard to that loan; however, the borrower must have a source of funds to make the payments. That source of funds itself has a cost, which reflects its own interest rate. If he uses other available funds to make the payments, the borrower is then unable to earn interest elsewhere on those funds. Or, if he borrows the funds to make the payments, the borrower must pay additional interest on such additionally borrowed funds. Thus, effectively, the funds cost interest for the entire year (unless the borrower steals or prints the currency, it has a cost). Stating the uncompounded periodic rate on the particular loan as a compounded effective rate reflects the reality that the funds cost interest for the entire year.

The nominal annual interest rate and the annual percentage rate on an installment loan will always be lower than the effective rate. As a result, financial institutions rarely, if ever, disclose or otherwise advertise the effective rate of a loan. This contrast with their eagerness to advertise the effective rate on a deposit. Federal law does not require disclosure of the effective rate. In fact it expressly requires prominent disclosure of the annual percentage rate, which is always a lower number on an installment loan (and which does not reflect the above described reality). Also, federal law expressly permits the disclosure of a “Comparative Index of Credit Cost” which has some characteristics of an effective rate, but which also is inevitably lower than the effective rate of interest.

4. Annual Percentage Rate. In credit transactions not involving points or some other fees, this is the nominal annual interest rate. Transactions involving points and some other fees have an annual percentage rate which reflects both the nominal rate and the compounded amortized effect of the points or other fees. Disclosure of this rate is required by federal law for most credit

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3 15 U.S.C. § 1637 (for open ended credit); § 1638 (for other credit transactions).

4 12 C.F.R. § 226.11.
In credit parlance, a “point” is equal to one percent of the principal amount loaned. Thus on a $100,000 loan, one point equals $1000 and two points equals $2000. On a $200,000 loan, one point equals $2000 and two points equals $4000. Institutions charge points for three general reasons.

**First**, the points - which are actually discounted interest - are not reflected in the nominal annual interest rate. As a result, the nominal rate is understated. While lenders must prominently disclose the annual percentage rate, which reflects the points, it can do so along with disclosure of the nominal rate. Thus, lenders hope borrowers will visualize the nominal rate as the true rate, rather than the more accurate and higher - and sometimes less prominent - A.P.R. or the most accurate and highest - and almost certainly undisclosed - effective rate.

**Second**, the points - if attributable to a home loan - are generally deductible by the borrower for federal income tax purposes. As a result, borrowers may benefit from having more of the interest deductible in the first year of the loan.

**Third**, the points are almost always non-refundable. They are paid - either with separate funds or by being withheld from the loan proceeds - at the time of the loan transaction. If the loan is outstanding for its entire term, the points are effectively paid periodically over the life of the loan. However, if the borrower pays the loan prematurely, he must pay all remaining principal, unreduced by the points. For example, a $100,000 loan with two points is the equivalent of a $98,000 loan because just as soon as the borrower receives the $100,000 he must pay back $2000 as points. Nevertheless, the borrower is immediately liable for the entire $100,000 loan principal, even if he were to repay the loan the next day. As a practical matter, most home loans are paid early because they contain a “due on sale” clause, accelerating them whenever the underlying security changes hands. Many purchasers of residential property sell the property - and thus pay off the respective loan early - prior to the end of the original loan term. As a result, the lender earns an extraordinary interest rate - higher even than the original effective interest rate. Often, much higher.

### B. Other Important Financial Terms

Several other important terms arise in relation to interest rates. They include: simple yield, yield, yield to maturity, and internal rate of return. At least the first two of these terms are not terms of art: their precise definitions may vary from user to user. Thus anyone using either of them in a legal context should provide a precise definition. Likewise, anyone coming across them in a legal context should demand a precise definition. In contrast, the latter two terms have generally accepted,
precise meanings.

a. **Simple Yield.** This is an easy-to-compute, but imprecise measure of the return on a debt instrument. It is the nominal annual interest divided by the current market price of the instrument.

<table>
<thead>
<tr>
<th>A $1,000 bond paying 7.0% nominal annual interest, paid quarterly - $70.00 per year or $17.50 per quarter - would have a simple yield of 7.0% if it sold for $1,000.00, a simple yield of 7.78% if it sold for $900.00, or a simple yield of 6.36% if it sold for $1,100.00.</th>
</tr>
</thead>
</table>

Example 1

This computation would change constantly, as the market value of the instrument changed. The ease of computation justifies the use of the figure. It is, however, an inferior measure of the true return on the bond. The actual yield for a stated period or the yield to maturity would be more accurate and thus more useful.

Some users may interchange this term with the slightly different term “yield.” Others might compound the quarterly payment to generate a more precise calculation. Neither use is wrong - they are merely different. As cautioned above, if someone uses the term “simple yield,” request a definition.

b. **Yield.** This measure of the return on a debt instrument is sometimes interchanged with the slightly different term “simple yield.” More commonly, however, it constitutes the actual yield on an instrument for a stated period of time. Thus it would divide any periodic interest payment by the purchase price and then convert it to an annual rate, compounding the periodic rate for the number of periods.

<table>
<thead>
<tr>
<th>A $1,000 bond paying 7.0% nominal annual interest, paid quarterly - $70.00 per year or $17.50 per quarter - would have a one-year yield of 7.186% if it sold for $1,000.00, a one-year yield of 7.98% if it sold for $900.00, or a one-year yield of 6.53% if it sold for $1,100.00.</th>
</tr>
</thead>
</table>

Example 2

This measure of the instrument differs from the “simple yield” in two respects. First, it adds the compounding feature, when appropriate; hence the yields are greater when the payment period is less than one year. Second, it does not change constantly as the market price of the instrument changes; instead, it is fixed by the purchase or issue price of the instrument (depending on whose viewpoint is involved).
Also, although the above definition describes a number which is more accurate and hence more useful than the number described as a “simple yield,” this term - yield - does not present a true measure of an instrument’s return. Two inaccuracies are inherent.

One, it relies on interest compounding, when, in fact, as far as the instrument is concerned the interest is paid and thus does not compound. This is less criticism of the calculation - and more mere observation, however, because that feature is inevitable. For yields to be useful, they must generally be comparable to those of other instruments. For this to be possible, they must be based on a common standard - such as the year. Instruments which pay interest annually will thus present an accurate yield. In contrast, instruments which pay at periods other than a year will never present an accurate annual yield: it violates the fourth rule stated earlier: the payment period and the compounding period must be the same. For instruments that pay interest other than annually, an annual yield will never be precise because it inherently requires an assumption that the interest paid continued to earn interest at the same internal rate. While useful, such an assumption is not perfect. As long as users understand this feature, the calculation of a yield can be very useful and generally accurate.

A second inaccuracy of the “yield” calculation involves its failure to consider the impact of changing values, i.e., changing market interest rates. Another way of stating this somewhat obvious point is that the term - as defined above - ignores market discounts and premiums. While the point of the calculation is simply to look at paid returns for a particular period - and thus it accomplishes what its definition constrains it to do, the calculation nevertheless risks presenting a significantly inaccurate picture. For example, an instrument sold at a premium will have the same “yield,” regardless of its life, it will have a higher “yield to maturity” the longer the period until maturity. Similarly, an instrument sold at a discount will have the same “yield,” regardless of its life, it will have a lower “yield to maturity” the longer the period until maturity. Despite some inherent inaccuracies of its own, the “yield to maturity” calculation present the most accurate and useful picture of a debt instrument. Hence a comparison of the yields of two instruments, ignoring the terms of the instruments, might (though not necessarily) present a small, or even largely distorted picture. A comparison of yields that considers the terms would indeed be mostly accurate; however, it would also be a comparison of “yields to maturity” and thus, by definition, not a comparison of mere yields.

c. Yield to Maturity. This is the most accurate measure of the return on a debt instrument. Comparable to - and sometimes interchanged with either the “effective interest rate” or the “internal rate of return” - it considers the instrument’s actual cash flows. Thus it is the most realistic measure of an instrument’s return.
The yield to maturity calculation amortizes the premium or discount element of the issue price over the life of the instrument. This is more useful than the mere “yield” which, as defined above, ignores the premium or discount. Nevertheless, the yield to maturity calculation is subject to at least two potential inaccuracies.

First, it assumes - as does the effective interest rate - that any payments continue to earn or cost the same constant interest rate. This is unlikely to be accurate; nevertheless, because no investor has a crystal ball with which to determine future investment returns, such an assumption is the best possible. It also permits realistic comparisons between instruments. Nevertheless, it can result in some misunderstandings and thus should be fully understood.

Example 3

A $1,000 bond paying 7.0% nominal annual interest, paid quarterly - $70.00 per year or $17.50 per quarter - and sold at par would have a yield to maturity of 7.186%, regardless of its life. If, instead, it sold for $900.00 and were outstanding for two years, it would have a yield to maturity of 13.36%. If outstanding for ten years, it would have a yield to maturity of 8.77%. If, instead it sold for $1,100 and were outstanding for two years, it would have a yield to maturity of 1.91%. If outstanding for ten years, it would have a yield to maturity of 5.68%.

The yield to maturity calculation amortizes the premium or discount element of the issue price over the life of the instrument. This is more useful than the mere “yield” which, as defined above, ignores the premium or discount. Nevertheless, the yield to maturity calculation is subject to at least two potential inaccuracies.

First, it assumes - as does the effective interest rate - that any payments continue to earn or cost the same constant interest rate. This is unlikely to be accurate; nevertheless, because no investor has a crystal ball with which to determine future investment returns, such an assumption is the best possible. It also permits realistic comparisons between instruments. Nevertheless, it can result in some misunderstandings and thus should be fully understood.

Example 4

A $100,000 face instrument issued for $96,000 and paying 10% nominal annual interest for five years will have a yield to maturity of 11.084585%. Also, $96,000 invested at 11.084585 nominal annual interest compounded annually for five years will generate $162,382.87. But, as shown in Chart 1, the original instrument will generate approximately $162,382.87 only if each $10,000 interest payment is itself reinvested at 11.084585% (the extra $1.01 is due to a rounding error).
The assumption that all returns are reinvested at the same rate, while necessary mathematically, can cause misunderstanding. An investor might assume that the two instruments in Example 4 are interchangeable because they have the same original cost and the same yield to maturity. Because they have different cash flows, however, they are comparable only with the above assumption, which may - or may not - be realistic.

The second potential inaccuracy involving the yield to maturity calculation involves the assumption that the instrument will be outstanding for its entire expected life. This, again, is a necessary assumption: to input a future maturity value one must know the future date. Because no crystal balls exist to foretell the future, the assumption becomes necessary that the instrument will continue to be outstanding for its entire scheduled life and will make all scheduled payments. Many instruments, however, have a put or call feature under which either the maker of purchaser - or both - may offer or demand payment early, respectively. In such cases, the statement of a yield to maturity should note the assumption regarding maturity.

d. Internal Rate of Return. This is the effective interest rate at which the initial investment equals the present value of all future cash flows. If all cash flows are level and in the same direction, this computation is relatively simple and essentially parallels the computation of a yield to maturity. Uneven cash flows - and particularly those which change direction - present computational difficulties. Most calculators actually use a trial and error approach because the formula can be extremely complex.