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Evaluation of a Model for the Evolution of Wear in a Scotch-Yoke Mechanism

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A nearly ideal two-dimensional scotch yoke mechanism was constructed to test a model of wear depth as a function cycle number. Model variables include the reciprocating mass, a two dimensional wear-rate, crank radius, and angular velocity. The model originally developed by T. A. Blanchet (1997), was nondimensionalized and simplified under conditions of large numbers of cycles, to predict the importance of including coupling based solely on a ratio of maximum allowable wear depth to the crank radius. Experiments show a linear progression of wear over two distinct regions, suggesting a sudden transition in wear modes just after 1.5 million cycles. The need for cycle or time dependent wear rates in analysis, which is a potentially far more significant source of error, is clearly illustrated by the experiment and discussed. [DOI: 10.1115/1.1537271]

Introduction

The ability to predict the useful life of a dynamic mechanism is of great importance to many designers and engineers. To accurately predict how a mechanism will perform over an extended number of cycles requires knowledge of how the components are changing during operation. A number of researchers have used numerical techniques to model wear in simulations of various contact problems and simple mechanisms [1–12]. Additionally, there are two closed form solutions known to the authors [13,14].

Generally, there is good agreement between the trends predicted by these numerical techniques, and the results gathered experimentally. The closed form expressions for the evolution in the components geometry and kinematics as a result of wear [13,14] are untested. Blanchet [13] published a two dimensional model for the simple harmonic oscillator known as the Scotch Yoke (a schematic is shown in Fig. 1). This mechanism is composed of a disk with a rigidly attached pin that slides within a vertical slot connected to a mass that slides within a frictionless guide. The mechanism produces a purely sinusoidal motion when the faces of the vertical slot are both straight and parallel. Equations (1)–(3) are the contact force $F_n(\theta)$, the wear depth $\Delta_n(\theta)$, and the position of the slider $X_n(\theta)$ respectively. These equations are in terms

of the number of crank revolutions n , the radius of the crank R (mm), the mass m (kg), the thickness of the slider b (mm), the angular coordinate θ , the angular velocity $\dot{\theta}$ (1/s), and a two-dimensional wear-rate k (mm^2/N), which is a wear depth per line load. The model is developed for a specific harmonic oscillator but is not tied to any particular material, although the wear on the pin is neglected.

$$F_n(\theta) = -m \dot{\theta}^2 R \cos(\theta) \left(1 + \left(\frac{k}{b} \right) m \dot{\theta}^2 \right)^{n-1} \quad (1)$$

$$\Delta_n(\theta) = R \cos(\theta) \left(\left(1 + \left(\frac{k}{b} \right) m \dot{\theta}^2 \right)^{n-1} - 1 \right) \quad (2)$$

$$X_n(\theta) = R \cos(\theta) \left(1 + \left(\frac{k}{b} \right) m \dot{\theta}^2 \right)^{n-1} \quad (3)$$

These equations were found through recursive application of equations for wear depth and contact force. Equation (4) gives the cyclic rate-of-change of the cyclic rate-of-change of the depth of wear ($\partial^2 |\Delta| / \partial n^2$).

$$\frac{\partial^2 \Delta_n}{\partial n^2} = R |\cos(\theta)| \left(1 + \frac{km \dot{\theta}^2}{b} \right)^{n-1} \ln \left(1 + \frac{km \dot{\theta}^2}{b} \right)^2 \quad (4)$$

Equation (4) is positive for positive values of crank radius, mass, thickness, angular velocity, and wear rate. Therefore, the rate-of-change of wear depth is a continuously increasing function with number of cycles, and predictions in wear made from first cycle loading and kinematics will under predict the actual amount of wear.

Nondimensionalization and Analysis

Equations (1)–(3) are nondimensionalized using the following dimensionless groups: load $F^* = F / (m \dot{\theta}^2 R)$, wear-depth $\delta^* = \Delta / R$, position $x^* = X / R$, and wear $\beta^* = m \dot{\theta}^2 k / b$.

$$F^*(\theta) = -\cos(\theta) (1 + \beta^*)^{n-1} \quad (5)$$

$$\delta^*(\theta) = \cos(\theta) \left((1 + \beta^*)^{n-1} - 1 \right) \quad (6)$$

$$x^*(\theta) = \cos(\theta) (1 + \beta^*)^{n-1} \quad (7)$$

Equation (8) is a nondimensional expression of wear-depth found by extrapolating the predicted wear-depth from the first cycle to n cycles. A ratio of the extrapolated prediction δ_e^* and the coupled prediction (Eq. (6)), is given in Eq. (9).

$$\delta_e^*(\theta) = \cos(\theta) \beta^* (n-1) \quad (8)$$

$$\frac{\delta_e^*}{\delta^*} = \frac{\beta^* (n-1)}{\left((1 + \beta^*)^{(n-1)} - 1 \right)} \quad (9)$$

The angular coordinate disappears in such a ratio (Eq. (9)) and the resulting expression can be plotted for various values of β^* versus the number of cycles n , see Fig. 2. For positive values of $\dot{\theta}$, β^* must be positive and greater than 0. The condition $\beta^* = 0$ corresponds to a condition of infinite wear resistance $k \rightarrow 0$, or no

Contributed by the Tribology Division for publication in the ASME JOURNAL OF TRIBOLOGY. Manuscript received by the Tribology Division April 9, 2002 revised manuscript received September 10, 2002. Associate Editor: S. Wu.

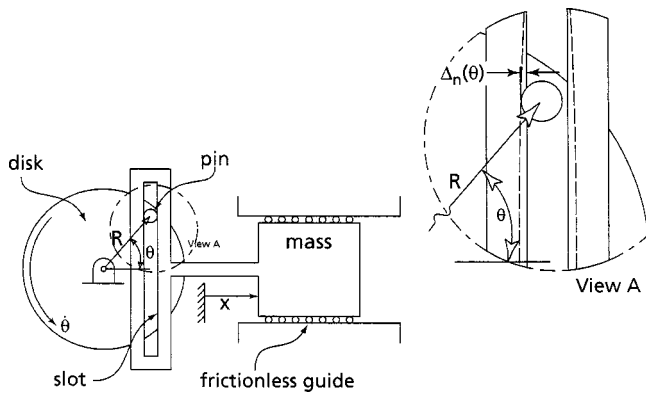


Fig. 1 A schematic of the two-dimensional scotch-yoke mechanism and the associated nomenclature

mass $m \rightarrow 0$, or infinite width $b \rightarrow \infty$, or a stationary crank $\dot{\theta} \rightarrow 0$, any of which result in zero wear depth. There appears to be no upper limit on β^* , although if $\beta^* = 1$ the maximum wear depth equals the crank radius after the first cycle. It appears from Fig. 2 that under conditions of either high β^* or high number of cycles the extrapolated prediction δ_e^* may greatly under predict the actual amount of wear on the mechanism.

Cycle dependent wear-rates can be expressed under the constraints of steady or constant mass, angular velocity, and width. Looking specifically at the maximum wear location, $\theta = 0$, Eq. (6) is rearrange to give β^* as a function of n .

$$\beta^* = (\delta_{(\theta=0)}^* + 1)^{(1/n-1)} - 1 \quad (10)$$

Using Eqs. (9) and (10) in concert, an expression for the ratio of the extrapolated prediction to the coupled prediction in terms of the coupled prediction and number of cycles is developed. Simplifying this expression for $n \gg 1$ gives Eq. (11), which is a weak function of n for $0 < \delta^* < 1$.

$$\begin{aligned} \frac{\delta_e^*}{\delta^*} &= \frac{((\delta^* + 1)^{(1/n-1)} - 1)(n-1)}{((1 + ((\delta^* + 1)^{(1/n-1)} - 1))^{(n-1)} - 1)} \\ &\cong \frac{((\delta^* + 1)^{(1/n)} - 1)(n)}{\delta^*} \quad (11) \end{aligned}$$

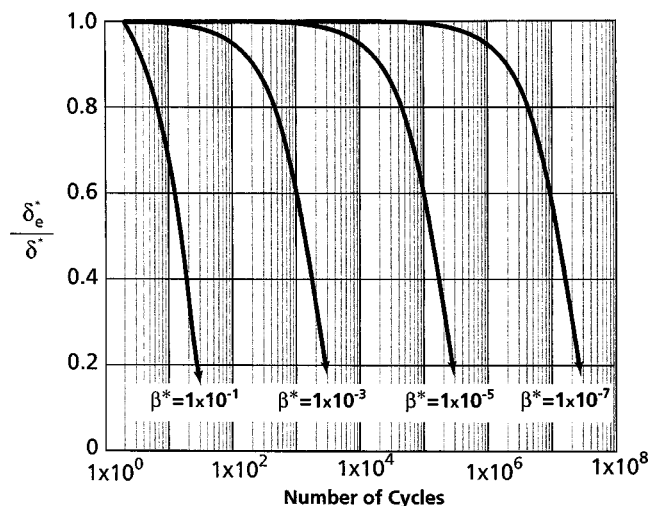


Fig. 2 Plot of the nondimensional ratio of the extrapolated prediction of wear-depth to the prediction of wear-depth made including the coupling between wear and load

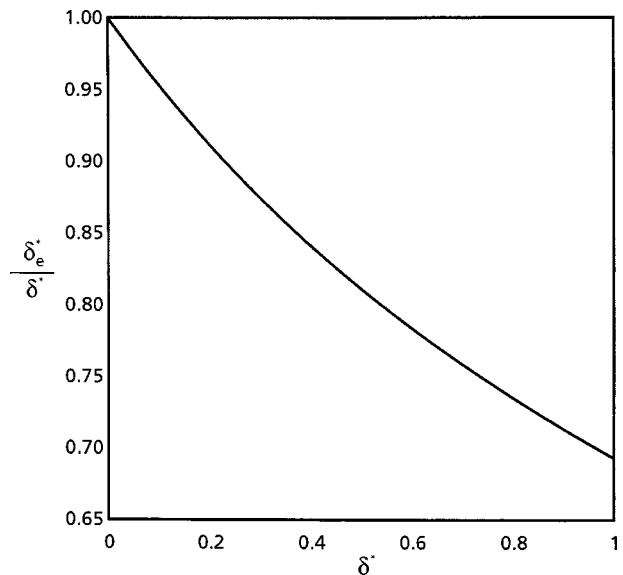


Fig. 3 Plot of the ratio of the extrapolated prediction in wear depth to the prediction that includes coupling versus the coupled prediction

Taking the limit of Eq. (11) as the number of cycles approaches infinity gives a simple expression for the error associated with ignoring coupling as a function of normalized wear depth at the $\theta = 0$ location. Figure 3 plots this function for $0 < \delta^* < 1$, which corresponds to a maximum dimensional wear depth range of $0 < \Delta < R$.

$$\frac{\delta_e^*}{\delta^*} = \lim_{n \rightarrow \infty} \frac{((\delta^* + 1)^{(1/n)} - 1)(n)}{\delta^*} = \frac{\ln(1 + \delta^*)}{\delta^*} \quad (12)$$

This analysis suggests that the effect of coupling for this simple dynamic mechanism is negligible under conditions where the maximum depth of wear is small relative to the crank radius.

Experimental

A scotch yoke mechanisms was designed and constructed to test this model. The requirements for this mechanism were: (1) a nearly frictionless platform for the mass to reciprocate on, (2) the kinematics need to be essentially two-dimensional, (3) the initial surfaces in the vertical need to be parallel and flat, and (4) a constant angular velocity. Additionally, it is desirable to continu-

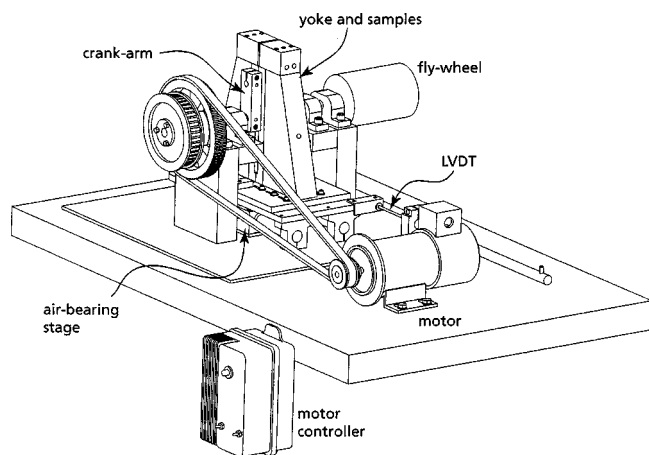


Fig. 4 A schematic of the experimental scotch-yoke apparatus

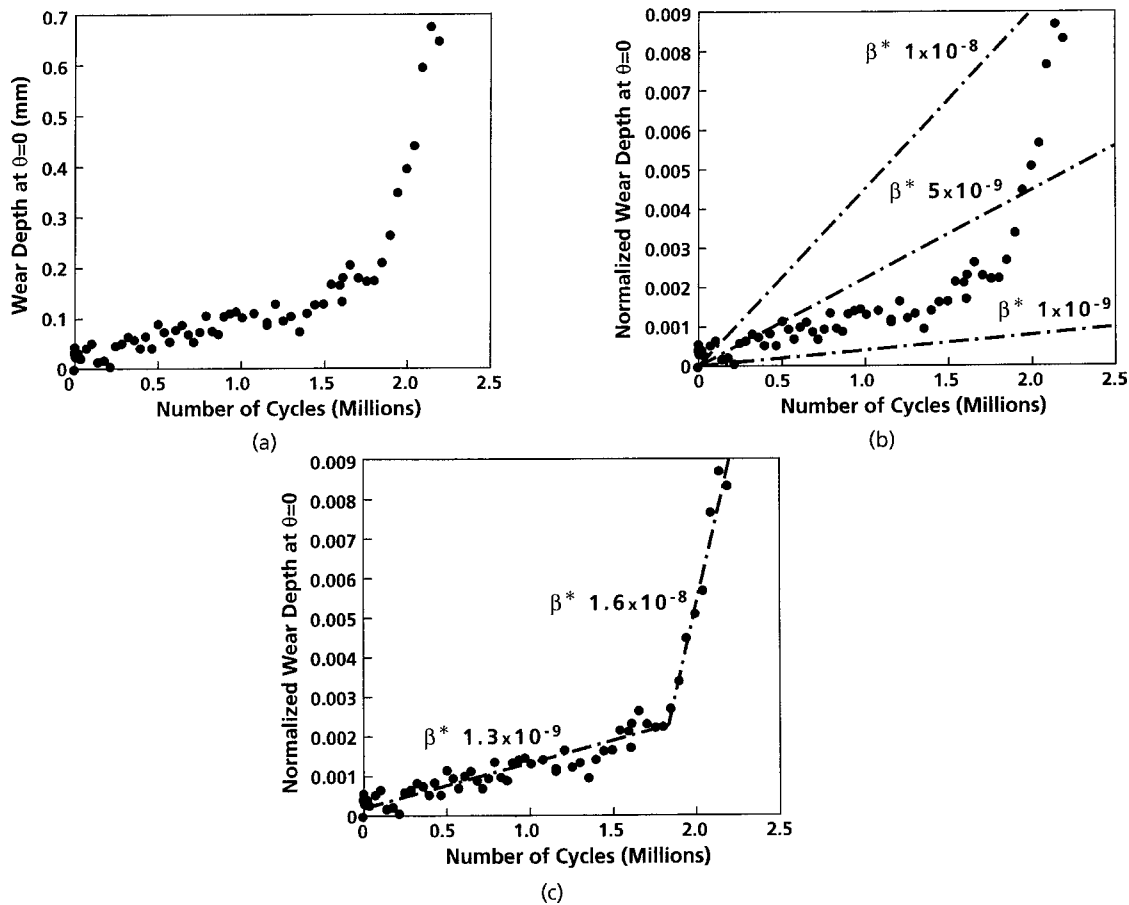


Fig. 5 (a) Plot of the depth of wear at $\theta=0$ versus, the number of cycles n as measured during the experiment (b) plot of the normalized depth of wear at $\theta=0$ versus the number of cycles n and compared to various prediction of the model given by Eq. (5) and (c) plot of the normalized depth of wear at $\theta=0$ versus the number of cycles n and compared to a prediction including a wear rate transition

ously monitor and measure the kinematics, and have access to the specimens for periodic inspection. A schematic of this apparatus is shown in Fig. 4.

The mechanism was constructed primarily from aluminum plate and steel shafts. The total reciprocating mass was 16.8 kg. The pin specimen was made from stainless steel rod 12.7 mm in diameter with an initial RMS roughness of $0.27 \mu\text{m}$. The rotating radius of the pin specimen was $R=7.88 \text{ cm}$. The slot specimen was made from commercially available High Density Polyethylene (HDPE) with a contact width $b=5.1 \text{ cm}$. The entire mass reciprocated on an air-bearing stage that was made from four 25.4 mm porous carbon air-bearings to provide as nearly frictionless and two-dimensional motion as possible.

The angular velocity of 10.5 radians per second was provided by a 0.37 kW DC motor with a 10:1 gear reducer and a 2:1 pulley and timing-belt system. The motor was controlled through a matched DC motor controller.

The kinematics were monitored using a roughly 20 cm Linear Variable Differential Transformer (LVDT) attached to the bottom of the reciprocating mass. The voltage was conditioned to give a full span of $\pm 5 \text{ VDC}$. This signal was read continuously on a computer data acquisition system with a 16bit A/D conversion ($\sim 3 \mu\text{m}$ discretization). Periodically a waveform was captured and saved to the computer.

Results

After processing the collected waveforms, the angular coordinate of $\theta=0$ was selected from each waveform and the difference in position from the first cycle position was reported as wear

depth $\Delta_{(n,\theta=0)} = X_{(n,\theta=0)} - X_{(1,\theta=0)}$, this is plotted in Fig. 5(a). Using the difference in displacement at the $\theta=0$ position as a measure of wear at the conclusion of the test was calculated to be 0.7 mm.

At the completion of the test, a freehand coordinate measuring machine was used to measure the wear-depth at the $\theta=0$ location. The coordinate measuring machine reported an average wear depth across the specimen at this location of $0.77 \text{ mm} \pm 0.1 \text{ mm}$. The sample was weighed at the end of the test and the mass lost was found to be 1.82 grams. Calculations of mass loss by integrating the recorded wear depth profile and multiplying this volume by the density of HDPE gives an expected mass loss of 1.95 grams.

Discussion

There is agreement between the measurements from the coordinate measuring machine, mass loss, and the kinematic data. Thus, the cycle dependent kinematic data is used for comparisons to the model. The data as compared to the model over a range of β^* is shown in Fig. 5(b). The model fit to the collected data is poor due to the transition in wear rate around 1.5 million cycles.

Using Eq. (12), and $\delta^* = 0.7 \text{ mm} / 78.8 \text{ mm} = 0.0089$, the error associated with neglecting coupling is less than 0.5 percent. The model predicts no significant coupling. It is suggested that the appearance of the transition is the result of a change in wear modes from mild to severe wear. This is supported by the observation of large flaky wear debris that was ejected in great quantities from the apparatus after 1.5 million cycles while very fine

powdery debris was observed prior to this. Curve-fitting the data set, it is readily apparent that a significant increase in wear rate occurred, as shown in Fig. 5(c).

Coupling may be important for some systems, however, the added difficulty of including coupling in numerical analysis and modeling may not significantly improve the predictions over the far simpler first cycle extrapolations. For some similar dynamic systems the nondimensional group δ^* , formulated as a wear depth over a characteristic throw length, and Eq. (12) may guide the decision to include or neglect coupling. Additionally, the need for cycle or time dependent wear rates in analysis, which is a potentially far more significant source of error, is clearly illustrated by the experiment.

Conclusions

1) The model for the coupled evolution for wear and load in the scotch yoke mechanism was non-dimensionalized.

2) The errors induced by ignoring coupling can be estimated by the ratio $\ln(1 + \delta^*)/\delta^*$.

3) The experiments showed a very linear progression in depth over 2 million cycles, with a single transition in wear depth occurring after ~ 1.5 million cycles. It is suggested that this is due to a transition from mild to severe wear.

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