TRIBOLOGY METHODS

Uncertainty in Pin-on-Disk Wear Volume Measurements Using Surface Scanning Techniques

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Abstract The uncertainty of wear volumes measured using surface scanning techniques is often neglected or assumed to be equivalent to the instrument error. A method is proposed that accounts for the number of wear volume scans, the variations in those scans, and the geometry of the experimental system as an improved measure of uncertainty. It demonstrates that the uncertainty in volume is directly correlated to the number of scans taken. A non-uniform wear track was used to validate the method, and the minimal and optimal number of scans was found.

Keywords Uncertainty · Profilometry · Wear track

Surface scanning techniques can be extremely precise in analyzing wear tracks to compute wear volumes; however, in many instances it is impractical to analyze an entire wear track using atomic force microscopy, interferometry, or other techniques [1, 2]. For this reason, tribologists commonly measure subsections of the wear track at evenly spaced intervals to compute wear volume, which leaves room for errors due to the variations in the wear track [3–6]. These errors are far more significant than the uncertainties of the instruments themselves. An useful measurement of uncertainty must consider the number of scans, variation of those scans, and the geometry of the experimental system.

The number of scans (N) required to accurately measure and predict the volume loss (V) of the wear track can be determined using uncertainty analysis. The volume loss of a wear track of nominal radius R with individually scanned cross-sectional areas (Ai) for a wear track subdivided into N sections of θi, where θi is held constant for each scan, which is shown in Fig. 1. This volume loss is approximated in Eq. 1:

\[ V \approx \sum_{i=1}^{N} A_i R \theta_i = R \theta_i \sum_{i=1}^{N} A_i = \frac{2\pi}{N} \sum_{i=1}^{N} A_i = \frac{R^2}{2} \sum_{i=1}^{N} A_i \]

(1)

The uncertainty of the volume loss (uV) is determined using the law of propagation of uncertainty as shown in Eq. 2:

\[ u_V^2 = \left( \frac{\delta V}{\delta A_i} u_{A_i}^2 \right)^2 + \left( \frac{\delta V}{\delta R} u_R^2 \right)^2 \]

(2)

\[ \approx \sum_{i=1}^{N} u_{A_i}^2 (R \theta_i)^2 + \left( \sum_{i=1}^{N} A_i \theta_i \right)^2 u_R^2. \]

Assuming the uncertainty in the area (uA) is equal for every scan, the summation of uA is defined in Eq. 3.

\[ \sum_{i=1}^{N} u_{A_i} = u_A N. \]

(3)

This definition is applied to Eq. 2 and multiplied by unity \( \left( \frac{N}{N} \right) \) yielding Eq. 4.

\[ u_V^2 \approx R^2 \theta_i^2 u_A^2 N \left( \frac{N}{N} \right) + \theta_i^2 \left( \sum_{i=1}^{N} A_i \right)^2 u_R^2 \left( \frac{N}{N} \right)^2. \]

(4)
The number of scans multiplied by the portion of the wear track analyzed is equal to the full revolution of \(2\pi\), which is shown in Eq. 5.

\[
\sum_{i=1}^{N} \theta_i \cdot \theta_i = 2\pi \cdot N.
\]

Equation 4 may now be rewritten as Eq. 6.

\[
\mu_V \approx \frac{(2\pi R)^2}{N} \cdot \frac{2}{R^2 N^2} \left( \sum_{i=1}^{N} A_i \right)^2.
\]

The definition of the standard deviation of the area can be manipulated to express the summation of the area in terms of the mean area \(\bar{A}\), standard deviation of the area \(\sigma_A\), and number of scans.

\[
\sigma_A^2 = \frac{(\sum A_i)^2}{N} - \bar{A}^2 \Rightarrow \left( \sum A_i \right)^2 = N(\sigma_A^2 + \bar{A}^2).
\]

The uncertainty in the area measurement due to instrument error is much smaller than the standard deviation of the measured areas; therefore, the standard deviation of the areas can be simplified to:

\[
\sigma_A^2 = \frac{(\sum A_i)^2}{N} - \bar{A}^2 \Rightarrow \left( \sum A_i \right)^2 = N(\sigma_A^2 + \bar{A}^2).
\]
deviation ($\sigma_A$) will be used in place of the uncertainty in area ($u_A$) to account for maximum error. The final expression for approximating uncertainty in volume is shown in Eq. 8.

$$u_V \approx \frac{2\pi R}{\sqrt{N}} \sqrt{\frac{\sigma_A^2}{2} + \frac{u_R^2}{R^2} (\sigma_A^2 + A^2)}.$$  (8)

As the number of scans increased, the estimated volume loss was closer to the true value and the associated uncertainty decreased. This is always valid, though increases in accuracy become negligible well before $N = 100$. For this extremely non-uniform wear track, four scans accurately estimated the volume loss 68% of the time, but with a large uncertainty value; Fig. 2b and c show eight scans yielded a better estimate. This method can be used in conjunction with other methods that compute wear rates and their uncertainties [7], and add validity to these types of measurements in scientific pursuits.

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References