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Wear 251 (2001) 1003–1008

WEAR

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Differential application of wear models to fractional thin films

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Abstract

Global application of bulk wear models, originally developed for monolithic bodies, to pre-deposited thin films and coatings can lead to the paradoxical prediction of wear removal rates exceeding the rate of film introduction into the tribological contact. A thin-film wear model is developed which resolves this paradox through differential wear model application, coupled with the fractional nature of such thin films which may exist as they are worn through and the corresponding fractional normal load that film supports. The model predicts the state of the wearing film as a function of position within the contact. A corresponding description of friction coefficient of contacts of such wearing fractional thin films is also developed, for purely sliding as well as combined rolling/sliding contacts. Furthermore, it is demonstrated that prediction of film state through the contact, from inlet to exit, enables subsequent prediction of the evolving global wear and friction behavior with time. The model is compared to examples of experimental friction data for thin films taken from the literature. The manuscript closes with a discussion of extension of the model to cases where such thin films are continuously replenished, such as in vapor phase lubrication. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Wear and friction modeling; Thin films and coatings; Environmental effects on friction; Vapor phase lubrication

1. Introduction

Wear behavior of a material is commonly characterized with quantities, such as dimensional wear coefficient K (m^3/Nm), deriving from an Archard model where wear volume increases linearly with product of normal load (F_n) and sliding distance. Wear coefficients are typically determined from tests, and subsequently utilized in applications, using bulk materials. However, extension of such wear models to thin solid films can prove paradoxical. For example, in a pin-on-coated disc contact with coating thickness h and wear track width w sliding at speed V, coating volume is being introduced into the contact at rate (Vwh). Global application of the wear model to the contact, however, would predict a rate of coating volume removal (KF_nV) that at high normal loads may exceed the rate of coating volume introduction, especially for thin films.

As a thin film wears, friction of the contact is often observed to go through a gradual transition. For example, in room temperature tests of thin (2 μm) silver coatings on alumina disks Erdemir et al. [1] reported an increase in coefficient of friction from an initial value of $\mu = 0.17$ towards an upper bound of $\mu = 0.6$ over a period of several hundred sliding cycles. Likewise, in the study of much thinner adsorbed films on silicon nitride, Ishigaki et al. [2] found friction coefficient to asymptotically approach $\mu = 0.8$ from an initial value of $\mu = 0.25$ over a period of several tens of cycles of repeated sliding. Describing similar adsorbed thin films Zaidi et al. [3] hypothesized the gradual variation of friction coefficient over a range of values to result from films only covering substrates fractionally, with upper or lower bounds of this range representing friction from bare substrate or continuous film. Consideration of this fractional nature of coverage, and therefore, the amount of the total normal load borne, by surface film will be important in tribological model development here. However, this aspect alone will not resolve the previously stated paradox, as the rate of film introduction into the contact and the potentially greater rate of film removal as predicted by global application of a wear model would both be reduced by the same fraction.

This paradox is resolved through applying wear models to the thin film differentially through the contact instead of globally. By this approach, an expression for film fractional coverage as a function of position within the contact and inlet fractional coverage will be developed, from which a rule-of-mixtures approach is derived to predict global friction coefficient. Considering that exit fractional coverage becomes inlet fractional coverage for the subsequent sliding cycle, evolution of friction, fractional coverage and, therefore, wear with time are also described for thin tribological films.

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0043-1648/01/$ – see front matter
PII: S0043-1648(01)00708-6

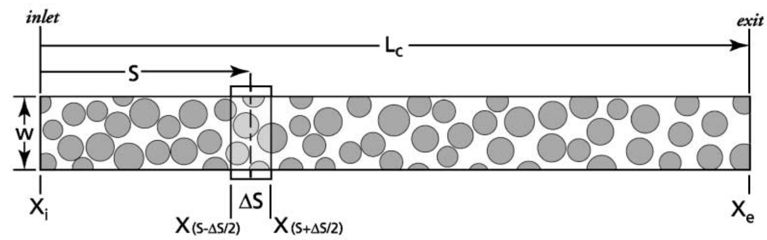


Fig. 1. One-dimensional surface model within a contact region of width w and length L_c, with surface film of fractional coverage $X(s)$ decreasing with increasing distance s through the contact due to wear, from X_i at the inlet to X_e at the exit.

2. Model development

2.1. Instantaneous pure sliding contact

A model of that portion of a wear track, fractionally covered by a film of thickness h and moving at speed V, instantaneously within a one-dimensional contact region against an uncoated stationary countersurface is illustrated in Fig. 1. The wear track has width w and the contact region has length L_c, with film fractional coverage $X(s)$ decreasing as a function of position s through the contact from X_i at its inlet to X_e at its exit due to film wear. Application of a wear model to a differential element of contact region of length Δs centered at position s would predict a volume rate of film wear $KX(s)p(s)w\Delta sV$, where $p(s)$ is contact pressure and, thus, $p(s)w\Delta s$ is the normal load upon the differential element such that $X(s)p(s)w\Delta s$ is the portion borne by the film. As shown in Eq. (1), this wear rate must balance the difference between the volume rates of introduction $whVX(s-\Delta s/2)$ and departure $whVX(s+\Delta s/2)$ of adherent film at either end of the differential element.

$$whVX\left(s-\frac{\Delta s}{2}\right) - whVX\left(s+\frac{\Delta s}{2}\right) = KX(s)p(s)w\Delta sV \tag{1}$$

Note that such differential application removes the thin-film wear paradox, since the rate of film wear will necessarily be less than the rate of film introduction into the differential element as Δs approaches zero. As $\Delta X = X(s+\Delta s/2) - X(s-\Delta s/2)$ is the change of fractional coverage over the element, Eq. (1) may be rearranged as a differential equation.

$$\frac{\mathrm{d}X}{X(s)} = -\frac{K}{wh}p(s)w\,\mathrm{d}s \tag{2}$$

Integration results in

$$\ln X(s) - \ln X(0) = -\frac{\kappa}{w}\int_0^s p(s')w\,\mathrm{d}s' \tag{3}$$

where s' is a dummy variable in this integration from the inlet ($s'=0$) to the position of interest (s), and $\kappa = K/h$ (units of m^2/Nm) is a modified wear coefficient representing area of thin film removed per product of sliding distance and normal load borne by the film. Eq. (3) may be rearranged as an explicit description of film fractional coverage.

$$X(s) = X_i \exp\left\{-\frac{\kappa}{w}\int_0^s p(s')w\,\mathrm{d}s'\right\} \tag{4}$$

where $X_i = X(0)$ is the inlet fractional coverage. Since the integral of $p(s')w\,\mathrm{d}s'$ through the entire contact to the exit ($s = L_c$) is by definition the total normal load F_n, despite the functionality of $p(s')$, the exit fractional coverage is always

$$X_e = X(L_c) = X_i\,e^{-F_n^*} \tag{5}$$

where $F_n^* = F_n\kappa/w$ is a representation of non-dimensional normal load.

With regards to friction modeling, while the film supports normal load $X(s)p(s)w\Delta s$ within the differential element, regions of bare substrate support $(1-X(s))p(s)w\Delta s$. Regions of bare substrate are characterized by friction coefficient μ_s, while μ_L characterizes regions where a lubricous film (or layer) is adherent. It is assumed that detached film debris are ejected from the contact and do not provide lubrication to regions of bare substrate as entrapped third bodies or countersurface transfer films. The applicability of this assumption will depend upon the specific tribosystem under study.

The friction force resulting from the differential element is the sum of contributions made by regions of lubricous film and bare substrate, each being their product of characteristic friction coefficient and normal load supported.

$$\Delta F_f = \mu_L X(s)p(s)w\Delta s + \mu_s(1-X(s))p(s)w\Delta s \tag{6}$$

Global friction force will be the integral of this quantity over the entire contact, and global friction coefficient is produced upon division by total applied normal load.

$$\mu = \frac{\int \mathrm{d}F_f}{F_n} = \frac{w}{F_n}\int_0^{L_c}(\mu_s - (\mu_s-\mu_L)X(s))p(s)\,\mathrm{d}s$$
$$= \mu_s - (\mu_s-\mu_L)\frac{w}{F_n}\int_0^{L_c}X(s)p(s)\,\mathrm{d}s \tag{7}$$

Though other one-dimensional pressure distributions $p(s)$, such as the semi-elliptical Hertzian profile may also be considered, the simplest case of uniform contact pressure

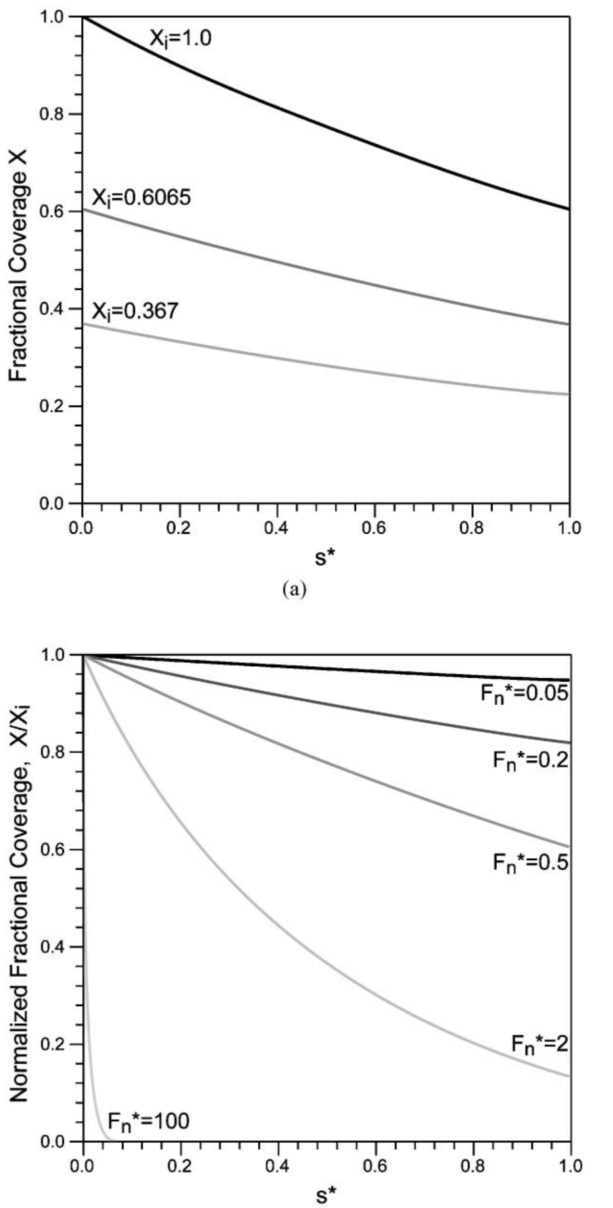


Fig. 2. Film fractional coverage X as a function of non-dimensional position $s^* = s/L_c$ within purely sliding contacts: (a) $X(s^*)$ for various inlet coverage X_i at $F_n^* = 0.5$; and (b) $X(s^*)/X_i$ for various non-dimensional normal loads F_n^*.

p is used here as an example. In such a case, Eq. (4) provides

$$X(s) = X_i \, e^{-F_n^* s^*} \tag{8}$$

where $s^* = s/L_c$ represents non-dimensional position through the contact. Fig. 2a and b depict distributions of fractional coverage of the wearing film through the contact as functions of inlet fractional coverage and normal load described by Eq. (8). With $X(s)$ available, Eq. (7) provides an expression for global friction

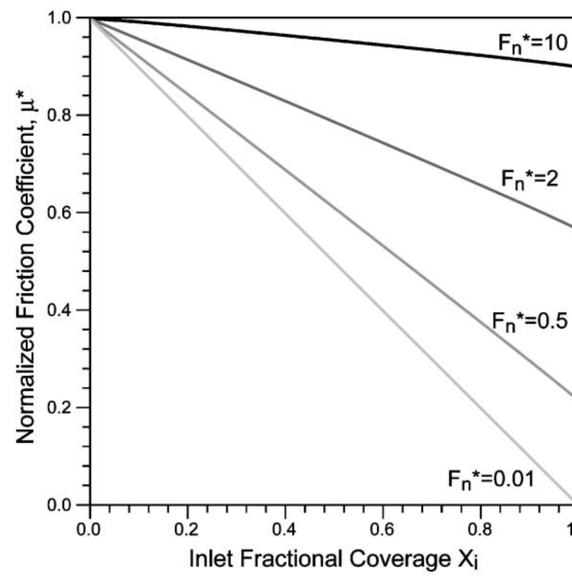


Fig. 3. Normalized friction coefficient μ^* as a function of inlet fraction coverage X_i for purely sliding contacts at various non-dimensional normal loads F_n^*.

coefficient

$$\mu = \mu_s - (\mu_s - \mu_L) X_i \frac{1 - e^{-F_n^*}}{F_n^*} \tag{9}$$

or in a normalized form

$$\mu^* = \frac{\mu - \mu_L}{\mu_s - \mu_L} = 1 - X_i \frac{1 - e^{-F_n^*}}{F_n^*} \tag{10}$$

with lower limit $\mu^* = 0$ representative of continuous thin film coverage and upper limit $\mu^* = 1$ representative of completely bare substrate.

Interestingly, friction coefficient is described as a function of normal load, in addition to inlet film fractional coverage, as also depicted in Fig. 3. The approach of μ^* towards its maximum value of 1 as normal load becomes large can be seen upon simple inspection of Eq. (10), and understood to result from nearly complete removal of the film within short distances beyond the inlet as seen in Fig. 2b. The approach of μ^* towards $(1 - X_i)$ as normal load becomes diminishingly small can be seen upon application of l'Hopital's rule to Eq. (10), and can be understood as $X(s)$ nearly remaining uniform at X_i throughout the contact (as also seen in Fig. 2b) due to negligible film wear. As a result μ^* should approach $(1 - X_i)$, the value of nearly uniform area fraction of the bare substrate leading to the higher friction.

2.2. Instantaneous combined rolling/sliding contact

A model of a one-dimensional combined rolling/sliding contact, with both surfaces possessing fractional thin surface films, is shown in Fig. 4. The two surfaces are distinguished as 'a' and 'b', with related quantities correspond-

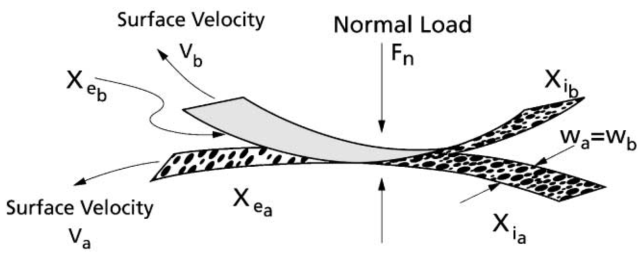


Fig. 4. Schematic view of wear tracks on two bodies (a and b) forming a combined rolling/sliding contact, with film fractional coverages at the contact exit X_e being reduced from those at the contact inlet X_i due to removal by wear.

ingly subscripted. As in the analysis of the purely sliding contact, a differential continuity analysis is performed on an incremental element of the contact region, balancing the difference in rates of introduction and departure of adherent film with the rate of removal by wear. Such a balance is performed for each body.

$$whV_aX_a\left(s-\frac{\Delta s}{2}\right)-whV_aX_a\left(s+\frac{\Delta s}{2}\right) = KX_a(s)p(s)w\Delta s(V_a-V_b) \quad (11a)$$

$$whV_bX_b\left(s-\frac{\Delta s}{2}\right)-whV_bX_b\left(s+\frac{\Delta s}{2}\right) = KX_b(s)p(s)w\Delta s(V_a-V_b) \quad (11b)$$

Note a convention of this model is that the higher speed surface is designated 'a', as wear rates are positive. Furthermore, it is presumed that the motion of each surface is along the same direction, so that $V_a > V_b > 0$.

As performed in the pure sliding case, the continuity analyses of Eqs. (11a) and (11b) are rearranged into differential equations from which expressions are generated for film fractional coverages as functions of position through the contact region.

$$X_a(s)=X_{i_a}\exp\left\{-\frac{\kappa}{w}\left(1-\frac{V_b}{V_a}\right)\int_0^s p(s')w\,ds'\right\} \quad (12a)$$

$$X_b(s)=X_{i_b}\exp\left\{-\frac{\kappa}{w}\left(\frac{V_a}{V_b}-1\right)\int_0^s p(s')w\,ds'\right\} \quad (12b)$$

Regardless of the functionality of the contact pressure distribution, integration of Eqs. (12a) and (12b) to the exit of the contact ($s = L_c$) always yields exit fractional coverage expressions

$$X_{e_a}=X_{i_a}\exp\left\{-F_n^*\left(1-\frac{V_b}{V_a}\right)\right\} \quad (13a)$$

$$X_{e_b}=X_{i_b}\exp\left\{-F_n^*\left(\frac{V_a}{V_b}-1\right)\right\} \quad (13b)$$

With regards to modeling friction, of the normal load $p(s)w\Delta s$ exerted on the incremental element, the fraction $\{X_a(s)+X_b(s)-X_a(s)X_b(s)\}$ is supported on film which may exist either on surface a or b, while the remaining fraction $\{1-(X_a(s)+X_b(s)-X_a(s)X_b(s))\}$ is supported where bare substrate areas on each surface contact one another. Thus, the friction force contributed by this incremental contact region is

$$\Delta F_f=\mu_L\{X_a(s)+X_b(s)-X_a(s)X_b(s)\}p(s)w\Delta s + \mu_s\{1-(X_a(s)+X_b(s)-X_a(s)X_b(s))\}p(s)w\Delta s \quad (14)$$

The total friction force would be the integral of this quantity, with global friction coefficient produced upon division by total normal load

$$\mu=\frac{\int dF_f}{F_n}=\mu_s-(\mu_s-\mu_L)\frac{w}{F_n}\int_0^{L_c}\{X_a(s)+X_b(s) - X_a(s)X_b(s)\}p(s)\,ds \quad (15)$$

For the case of uniform contact pressure p, integration of Eqs. (12a) and (12b) yields expressions for distributions of film fractional coverage of each surface.

$$X_a(s)=X_{i_a}\exp\left\{-F_n^*\left(1-\frac{V_b}{V_a}\right)s^*\right\} \quad (16a)$$

$$X_b(s)=X_{i_b}\exp\left\{-F_n^*\left(\frac{V_a}{V_b}-1\right)s^*\right\} \quad (16b)$$

With film fractional coverage distributions in place, an expression for friction coefficient for this example case of uniform contact pressure may be produced by performing the integral in Eq. (15)

$$\mu^*=1-\left(\frac{X_i^{[a]}(1-e^{-\kappa(F_n/w)((1-(V^{[b]}/V^{[a]}))})}{\kappa(F_n/w)(1-(V^{[b]}/V^{[a]}))} + \frac{X_i^{[b]}(1-e^{-\kappa(F_n/w)((V^{[a]}/V^{[b]})-1)})}{\kappa(F_n/w)((V^{[a]}/V^{[b]})-1)} - \frac{X_i^{[a]}X_i^{[b]}(1-e^{-\kappa(F_n/w)((V^{[a]}/V^{[b]})-(V^{[b]}/V^{[a]}))})}{\kappa(F_n/w)((V^{[a]}/V^{[b]})-(V^{[b]}/V^{[a]}))}\right) \quad (17)$$

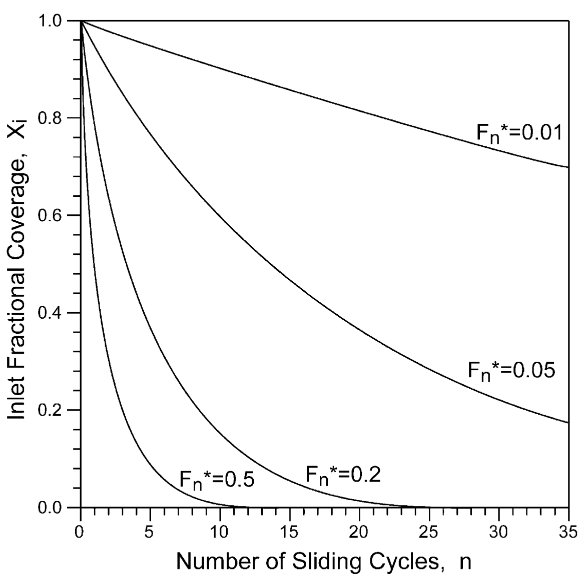
Note setting $X_i^{[b]}$ and V_b both equal zero, all equations for fractional coverage of surface 'a' as well as resultant friction coefficient from this combined rolling/sliding analysis revert to those from the previous analysis of purely sliding contact.

2.3. Time-dependent analysis

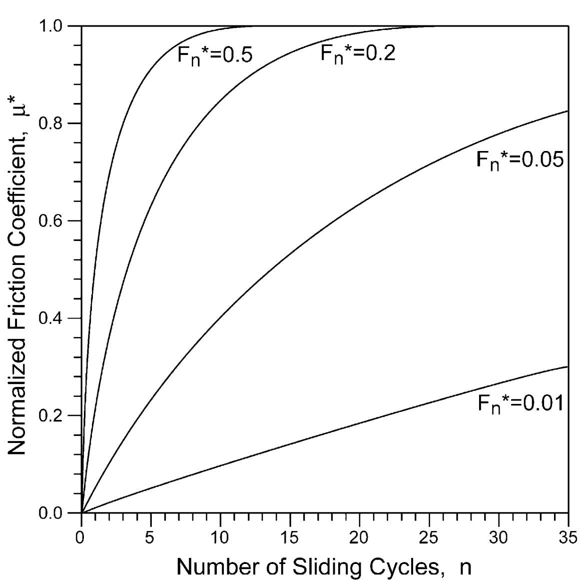
To predict evolution of fractional coverage of thin films due to wear, as well as resultant evolution of friction, one simply has to consider that exit fractional coverage from one cycle becomes inlet fractional coverage for the next. For the case of purely sliding contact, such evolving fractional coverage can be illustrated by the contours selected within

Fig. 2a for an example with $F_n^* = 0.5$. Fractional coverage which first enters the inlet with $X_i = 1$ exits with $X_e = 0.6065$. When this returns to the contact as $X_i = 0.6065$ the subsequent cycle further reduces it to $X_e = 0.367$. The next cycle produces $X_e = 0.222$, and so forth. From Eq. (5), the inlet fractional coverage for any arbitrary *j*th cycle may be determined from its value during the previous sliding cycle.

$$\frac{X_i(j)}{X_i(j-1)} = e^{-F_n^*} \tag{18}$$



(a)



(b)

Fig. 5. (a) Evolution of (a) inlet film fractional coverage $X_i(n)$; and (b) normalized friction coefficient $\mu^*(n)$, with increasing sliding cycles *n* for a purely sliding contact with initially continuous surface film ($X_i(0) = 0$) at various non-dimensional normal loads F_n^*.

Thus the evolution of inlet fractional coverage from its initial value $X_i(0)$ for the zeroth cycle to $X_i(n)$ for the *n*th cycle can be developed as

$$\begin{aligned} X_i(n) &= X_i(0)\left(\frac{X_i(1)}{X_i(0)}\right)\left(\frac{X_i(2)}{X_i(1)}\right)\cdots\left(\frac{X_i(j)}{X_i(j-1)}\right)\cdots \\ &\quad \left(\frac{X_i(n-1)}{X_i(n-2)}\right)\left(\frac{X_i(n)}{X_i(n-1)}\right) \\ &= X_i(0)(e^{-F_n^*})^n = X_i(0)\, e^{-nF_n^*} \end{aligned} \tag{19}$$

Substitution into Eq. (10) produces an expression for the time-dependent friction coefficient, specific to the case of a purely sliding contact of uniform contact pressure.

$$\mu^*(n) = 1 - X_i(0)\frac{1 - e^{-F_n^*}}{F_n^*}\, e^{-nF_n^*} \tag{20}$$

This evolution of fractional coverage and resultant friction coefficient for sliding contacts with initially continuous ($X_i(0) = 1$) thin films is displayed in Fig. 5a and b, respectively, for various normal loads. In each case X_i approaches zero and μ^* approaches unity asymptotically with increasing numbers of sliding cycles. Response for initial fractional coverages less than unity are described by considering only regions of Fig. 5a and b to the right of the *X*-axis location corresponding to that desired initial fractional coverage.

3. Discussion and closing remarks

Consideration of the potential fractional nature of thin tribological films coupled with differential wear model application enables description of removal that is non-paradoxical (predicted wear rate does not exceed the rate of material introduction into the contact). It also enables description of friction that gradually transforms from that of the film to that of the bare substrate, as has been observed experimentally. For example, the shape of the friction record for silver film under 10 N load at room temperature presented by Erdemir et al. [1] is similar to those predicted and presented in Fig. 5b, and would correspond to wear behavior approximated by $F_n^* = F_n\kappa/w = 0.01$. The shape of the friction record for the adsorbed films measured by Ishigaki et al. [2] under repeated sliding at 0.25 N load more nearly matches the predicted behavior for $F_n^* = F_n\kappa/w = 0.1$. In comparing these values, it must be kept in mind that $\kappa = K/h$, and that the adsorbed films have extremely small thickness *h*.

Regarding the studies of Ishigaki et al. [2], it should be considered that surface films are being adsorbed from the surrounding environment, and that replenishment of adsorbed film may proceed simultaneously with film removal by wear. Though this replenishment may be negligible in cases of immediately repeated sliding cycles, as dwell periods of increased duration were interposed between sliding cycles, friction increased less rapidly with increasing sliding cycles and settled to lower steady-state values. In light of concepts considered here, such steady-state friction coefficients intermediate those for continuous thin film and bare substrate (μ_L and μ_s, respectively) may be interpreted to result from stabilized non-zero inlet fractional coverages that are restored each sliding cycle before re-entering the contact region. Such replenishment of surface film may either be purposeful as in vapor phase lubrication where desired film precursors are intentionally admixed into the contact's surrounding environment, or passive as in the case of adsorption of pre-existing species such as watervapor.

Under steady conditions, which in addition to steady-state friction coefficient would have corresponding values of steady-state inlet and exit film fractional coverages X_i and X_e, the volume rate of film replenishment on wear track areas outside of the contact must balance the rate of film removal due to wear within the contact, $wh(X_i - X_e)V$. Zaidi et al. [3] presented an earlier model describing replenishment of fractional adsorbed oxygen surface films and resultant friction, however, no wear model was employed but instead the simplifying assumption was made that coverage was always completely reduced to zero within the contact prior to exit. Coupled with differential wear models detailed here, specifically Eqs. (13) and (17), Sawyer and Blanchet [4] have further developed such adsorption and vapor phase lubrication models that consider replenishment of thin fractional lubricous surface films. There are also instances where adsorbed films such as water will instead cause friction to rise, with extent of this increase dependent on the adsorption time elapsed since last contact, such as reported for diamond-like carbon [5] as well as Si-based MEMS devices [6]. Vapor phase lubrication models may also be applicable, though in these cases ($\mu_L > \mu_s$) normalized friction coefficients $\mu^* = 1$ and $\mu^* = 0$ will instead simply represent limits of low and high friction, respectively.

Acknowledgements

This work was supported in part by the National Science Foundation, under Grant no. CMS-9457596 as a Young Investigator Award. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

[1] A. Erdemir, G.R. Fenske, F.A. Nichols, R.A. Erck, Solid Lubrication of Ceramic Surfaces by IAD-Silver Coatings for Heat Engine Applications, STLE Preprints, 1989, 89-TC-2E-1.

[2] H. Ishigaki, R. Nagata, M. Iwasa, Effect of adsorbed water on friction of hot-pressed silicon nitride and silicon carbide at slow speed sliding, Wear 121 (1988) 107–116.

[3] H. Zaidi, D. Paulmier, J. LePage, The influence of the environment on the friction and wear of graphitic carbons. II. Gas coverage of wear debris, Appl. Surf. Sci. 44 (1990) 221–233.

[4] W.G. Sawyer, T.A. Blanchet, Vapor-Phase Lubrication in Combined Rolling and Sliding Contacts: Modeling and Experimentation. ASME Preprints, 2000-TRIB-39 (2000) (also accepted for publication in ASME J. Tribol.).

[5] J.A. Heimberg, K.J. Wahl, I.L. Singer, A. Erdemir, Superlow friction behavior of diamond-like carbon coatings: time and speed effects, Applied Physics Letters 78 (2001) 2449–2451.

[6] S.T. Patton, W.D. Cowan, J.S. Zabinski, Performance and reliability of a new MEMS electrostatic lateral output motor, in: Proceedings of the 37th Annual IEEE International Reliability Physics Symposium, 1999, pp. 179–188.