Computation of energy imparted in diagnostic radiology

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Energy imparted is a measure of the total ionizing energy deposited in the patient during a radiologic examination and may be used to quantify the patient dose in diagnostic radiology. Values of the energy imparted per unit exposure-area product, \( \omega(z) \), absorbed by a semi-infinite water phantom with a thickness \( z \), were computed for x-ray spectra with peak x-ray tube voltages ranging from 50–140 kV and with added filtration, ranging from 1–6 mm aluminum. For a given phantom thickness and peak x-ray tube voltage, the energy imparted was found to be directly proportional to the x-ray beam half-value layer (HVL) expressed in millimeters of aluminum. Values of \( \omega(z) \) were generated for constant waveform x-ray tube voltages and an anode angle of 12°, and were fitted to the expression \( \omega(z) = \alpha \times \text{HVL} + \beta \). Fitted \( \alpha \) and \( \beta \) parameters are provided that permit the energy imparted to be determined for any combination of tube voltage, half-value layer, and phantom thickness from the product of the entrance skin exposure (free-in-air) and the corresponding x-ray beam area. The results obtained using our method for calculating energy imparted were compared with values of energy imparted determined using Monte Carlo techniques and anthropomorphic phantoms for a range of diagnostic examinations. At 60, 80, and 120 kV, absolute values of energy imparted obtained using our method differed by 8%, 10%, and 12%, respectively, from the corresponding results of Monte Carlo computations obtained for an anthropomorphic phantom. The method described in this paper permits a simple determination of energy imparted for any type of diagnostic x-ray examination which may be used to compare the radiologic risks from differing types of x-ray examinations, optimize imaging techniques with respect to the patient dose, or estimate the patient effective dose equivalent. © 1997 American Association of Physicists in Medicine. [S0094-2405(97)01304-7]

Key words: integral dose, energy imparted, waveform ripple, anode angle, patient dosimetry

I. INTRODUCTION

Although the entrance skin exposure has been a popular method of expressing patient radiation doses, this parameter does not take into account the x-ray beam quality [i.e., half-value layer (HVL)] or the size of the x-ray beam. As a result, the patient exposure is generally a poor indicator of the risk associated with a given radiologic examination. In recent times, the effective dose equivalent, \( H_E \), and the effective dose, \( E \), have been used to quantify the dose to patients undergoing radiologic examinations. The major benefit of using the effective dose is that this parameter accounts for the absorbed doses and relative radiosensitivities of the irradiated organs in the patient and, therefore, better quantifies the patient risk. Unfortunately, the effective dose is difficult to determine because it generally requires a knowledge of the mean absorbed dose to each irradiated organ in the body. The energy imparted to the patient, also known as the integral dose, is a measure of the total ionizing energy deposited in the patient during a radiologic examination and may be used to quantify the patient dose in diagnostic radiology. In general, energy imparted is easier to calculate or measure than the effective dose. The energy imparted to a patient may be used directly as an approximate indicator of the patient risk or used to estimate the corresponding values of the effective dose equivalent.

The energy imparted during a radiologic examination may be obtained from measured depth dose data and methods derived from this approach. Energy imparted generally depends on the x-ray beam quality, as well as the field size and irradiation geometry, which makes depth dose data of limited value in the everyday clinical setting. Monte Carlo techniques may also be used to obtain values of energy imparted, but these methods are computer intensive, time consuming, and relatively cumbersome to use. The most practical approach developed to date to obtain values of energy imparted is the use of transmission ionization chambers, which can generate energy imparted data from an exposure-area, or air collision kerma-area product. Measurements of exposure-area product have been reported to result in an accuracy of energy imparted between 10% and 20%. However, exposure-area product meters do not take into account the patient thickness, and the incident beam may not totally irradiate the patient. Although it may be possible to overcome both of these limitations, an accurate and practical method for estimating energy imparted to patients that does not rely on special instrumentation would clearly be advantageous.

In this study, we describe a method to obtain an estimate of the energy imparted to patients undergoing radiologic examinations, which may be used with the dosimetry equipment available in most radiology departments. The method is
based on Monte Carlo calculations of energy imparted from monoenergetic photons and makes use of published diagnostic energy x-ray spectra. The patient is modeled as a homogenous slab of water with a specified thickness. The water equivalence of a given patient may be obtained by a semi-empirical model developed by Tucker et al. was used to compute x-ray spectra generated at tube voltages ranging from 50 to 140 kV using x-ray tube filtration ranging from 1 to 6 mm aluminum (Al). The generated x-ray spectra consisted of discrete energy bins, \( \Phi(E) \), each giving the number of x-ray photons per unit area at a given distance from the source in the energy interval between \( E-1 \) and \( E \) keV.

The waveform ripple factor, \( f \), of an x-ray generator may be expressed as

\[
f = \frac{V_p - V_i}{V_p},
\]

where \( V_p \) and \( V_i \) are the maximum (peak) and minimum x-ray tube voltages. The effect of a varying x-ray tube voltage on the output x-ray spectrum was accounted for by using the idealized waveform ripple shown in Fig. 1. The value of the minimum \( V_i \) was rounded to the nearest integer, \( V_i' \), and the voltage range between \( V_i' \) to \( V_p \) was subdivided into equal increments of 1 kV. X-ray spectra were generated at each voltage increment and were summed to yield the x-ray spectrum for a given waveform. The time intervals, \( \delta(t) \), during which the tube voltage increases from \( V_i \) to \( V_{i+1} \) (Fig. 1) was obtained using the expression

\[
\delta(t_i) = \sin^{-1}\left( \frac{V_{i+1}}{V_p} \right) - \sin^{-1}\left( \frac{V_i}{V_p} \right).
\]

The total time, \( \Delta(t) \), for the voltage to increase from \( V_i' \) to \( V_p \) was obtained using the expression

\[
\Delta(t) = \sin^{-1}\left( \frac{V_p}{V_p} \right) - \sin^{-1}\left( \frac{V_i'}{V_p} \right).
\]

The x-ray spectrum for a given waveform ripple was obtained using the expression

\[
\Phi(E) = \sum_{i=V_i'}^{V_p} \frac{\delta(t_i)}{\Delta(t)} \Phi_i(E),
\]

where \( \Phi_i(E) \) is the x-ray spectrum generated at a voltage \( V_i \). Equation (4) was used to generate x-ray spectra for x-ray tube voltage ripple factors ranging from 0% (i.e., constant potential) to 100% (i.e., single phase).

X-rays generated at a depth inside the target undergo self-attenuation in the target as a function of the tube anode angle that was employed in this study to obtain x-ray spectra for anode angles ranging from 6° to 20°.

### B. Entrance exposure and HVL

The exposure in air for an x-ray spectrum with discrete energy bins, \( \Phi(E) \), generated at a tube voltage \( V \) was obtained using the expression

\[
X = \frac{e}{W} \sum_{E=1}^{V} \frac{\mu_{ab}(E)}{\rho_{air}} E \Phi(E) \frac{C}{kg},
\]

where \( e \) is the unit charge, \( W \) is the ionization energy of air, and \( \mu_{ab}(E)/\rho_{air} \) is the mass energy absorption coefficient of air. The ionization energy of air was taken to be 33.97 eV per ion pair. The exposure \( X \) in C/kg was converted to roentgens using the relationship 1.0 \( R = 2.58 \times 10^{-4} \) C/kg. The HVL of an x-ray spectrum generated at a tube voltage \( V \) was obtained using an iterative method by varying the aluminum filtration thickness until the resultant x-ray beam exposure was reduced to 50% of the original value. The iterative process of computing the HVL resulted in an uncertainty of the HVL of no more than 0.01 mm.
C. Energy imparted: Water phantom

Figure 2 shows the irradiation geometry assumed for the computation of energy imparted. For a semi-infinite water phantom of thickness $z$ cm, irradiated normally by an x-ray spectrum generated at an x-ray tube voltage $V$, the energy imparted due to a uniform x-ray beam with 1 cm$^2$ cross-sectional area may be obtained from the expression

$$e(z) = \sum_{E=1}^{V} E (E, z) \Phi(E) \frac{J}{cm^2},$$

where $e(E, z)$ is the average energy in joules imparted to the water phantom with thickness $z$ cm by a normally incident monoenergetic photon of energy $E$. Polynomial expressions from Boone$^{19,24}$ were used to compute $e(E, z)$ over the energy range of 10–145 keV at increments of 1 keV.

For any diagnostic x-ray spectrum, the energy imparted per unit exposure-area product, $\omega(z)$, is the energy imparted to the phantom of thickness $z$, for an x-ray beam with a cross-sectional area of 1 cm$^2$, normalized to unit exposure (free-in-air) at the phantom surface with no backscatter. Values of energy imparted per unit area-exposure product, $\omega(z)$, were obtained by dividing Eq. (6) by Eq. (5), yielding

$$\omega(z) = \frac{W}{e} \frac{\sum_{E=1}^{V} E (E, z) \Phi(E)}{\sum_{E=1}^{V} \left( \frac{\mu_{eb}}{\rho_{au}} \right)_E E \Phi(E)} \frac{J}{R cm^2}.$$ 

Values of energy imparted per unit area-exposure product were computed from x-ray spectra generated at constant voltages, ranging from 50 to 140 kV using a tube anode angle of 12°. Added tube filtration ranged from 1 to 6 mm aluminum in increments of 0.5 mm. Computations of $\omega(z)$ were made for water phantom thicknesses ranging from 5 to 30 cm in increments of 5 cm.

D. Energy imparted: Anthropomorphic phantom

The results of energy imparted computations for representative radiologic examinations using the method described in this study were compared with corresponding energy imparted values determined from dosimetry data based on Monte Carlo calculations performed on an anthropomorphic phantom.$^{7}$ These Monte Carlo data provide values of the mean doses to three body regions [i.e., head (5.8 kg), trunk including arms (43.0 kg), and legs (22.1 kg)],$^{25}$ which permit the energy imparted to each body region to be determined from the product of the mean dose and mass of the specified body region. The energy imparted, $e$, to the anthropomorphic phantom was computed using

$$e = D_h m_h + D_t m_t + D_l m_l,$$

where $D_h$, $D_t$, and $D_l$ denote the average absorbed doses in the head, trunk, and leg regions, respectively, and $m_h$, $m_t$, and $m_l$ are the corresponding masses of these body regions.

Anterio–posterior and lateral view examinations covering the head, chest, stomach, and rectum were investigated. The published dosimetry data of Hart et al.$^7$ are normalized to unit air dose-area product (free-in-air) and the relationship that an exposure of 87.7 R results in an air dose of 1.0 Gy was used to convert air absorbed dose values to exposure. Typical entrance skin exposures (free-in-air) were assumed for each examination (listed in Fig. 8) with the x-ray beam cross-sectional area at the patient midplane obtained from

![Figure 2. X-ray irradiation geometry of a semi-infinite water phantom with a thickness of $z$ cm.](image1)

![Figure 3. Values of $\omega(z)$ (mJ R$^{-1}$ cm$^{-2}$) as a function of photon energy of monoenergetic photons for water phantoms with thicknesses from 5 to 30 cm.](image2)

![Figure 4. Values of $\omega(z)$, computed using Eq. (7), for a constant x-ray tube potential 3 mm Al filtration and an anode angle of 12° plotted as a function of water phantom thickness.](image3)
the published data of Hart et al.\textsuperscript{7} Inverse square law corrections were applied, using the specified SID/SSD data, to obtain the x-ray beam area at the patient entrance and appropriate corrections were incorporated for the fraction of the x-ray beam that intercepted the patient.\textsuperscript{26}

\section*{III. RESULTS}

Figure 3 shows the computed values of energy imparted per unit exposure-area product, obtained using Eq. (7), for monoenergetic photons as a function of photon energy. Values of $\omega(z)$ always increased with increasing water phantom thickness. For a given water phantom thickness, the $\omega(z)$ parameter initially increased before reaching an approximately constant plateau value. Figure 4 shows $\omega(z)$ computed using Eq. (7) as a function of phantom thickness for x-ray spectra generated at constant x-ray tube voltages ranging from 60 to 140 kV and an anode angle of 12°. The

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Values of $\omega(z)$ as a function of HVL for a water phantom thickness of 20 cm.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Values of $\omega(z)$ as function of the HVL for a water phantom of 20 cm.}
\end{figure}
increase of $\omega(z)$ with phantom thickness is largest at the smallest phantom thicknesses. For a phantom thickness of 20 cm at 80 kV, variations of phantom thickness of $\pm 0.5$ cm would change $\omega(z)$ by only $\pm 0.5\%$. At the same kV, however, a variation of $\pm 0.5$ cm for a nominal phantom thickness of 5 cm would change $\omega(z)$ by $\pm 7.0\%$.

Figure 5 shows values of $\omega(z)$ plotted as a function of the HVL for voltage waveform ripples between 0% and 100%. For a given x-ray tube voltage, $\omega(z)$ increased linearly with the HVL. At a given tube voltage and HVL, the waveform ripple had a minor impact on the value of $\omega(z)$. At 60 kV, going from a constant potential to a single phase generator reduced the value of $\omega(z)$ by less than 4%, and at 140 kV, this change was less than 1%. For modern x-ray generators...
with ripple factors, ranging from 0% to 20%, neglecting any waveform ripple when computing a value of $\omega(z)$ would result in errors of between 0% and 2% for any given kV and HVL. Figure 6 shows values of $\omega(z)$ plotted as a function of the HVL for anode angles between $6^\circ$ and $20^\circ$. For a given x-ray tube voltage, $\omega(z)$ increased linearly with the HVL. At a given tube voltage and HVL, the anode angle had a minor impact on the value of $\omega(z)$, and changing the anode angle from $6^\circ$ to $20^\circ$ affected the value of $\omega(z)$ by less than 2%.

Figure 7 shows the values of $\omega(z)$ computed using Eq. (7) for constant x-ray tube voltages and an anode angle of $12^\circ$ plotted as a function of the HVL. At a given tube voltage and phantom thickness, $\omega(z)$ increased linearly with the HVL. For each tube voltage and phantom thickness, values of $\omega(z)$ were fitted to the equation

$$\omega(z) = \alpha \times \text{HVL} + \beta = \text{J R}^{-1} \text{cm}^{-2}$$

(9)

where $\alpha$ and $\beta$ are parameters of the fit. Table I lists the resultant values of $\alpha$ and $\beta$ parameters for each listed kV and phantom thickness. All the correlation coefficients, $r^2$, obtained for these linear fits ranged between 0.984% and 1.00. All values of $\omega(z)$ computed using the $\alpha$ and $\beta$ parameters listed in Table I agree to within 1.9% when compared to the exact values obtained using Eq. (7).

Figure 8 shows values of energy imparted as a function of tube voltage for four body regions and two projections with an x-ray tube filtration of 3 mm Al, where the data points have been fitted to a second-order polynomial. In each figure, the water equivalent phantom assumed in our computation is provided with the error bars indicating the variations in energy imparted obtained by changing the water phantom thickness by ± 2 cm. For water phantom thickness values not listed in Table I, the appropriate $\omega(z)$ values were obtained from a polynomial fit of $\omega(z)$ plotted as a function of phantom thickness. For the same entrance exposure, values of energy imparted increased with increasing tube voltage and filtration. At 60 kV, the average difference between the two methods of determining energy imparted was 8% with a maximum difference of 20% for the LAT Stomach examination. At 80 kV, the average difference between the two methods was 10% with the maximum being 25% for the LAT Stomach examination. At 120 kV, the two methods produced results with an average difference of 12%, with the LAT Stomach examination resulting in a maximum difference of 29%.

### IV. DISCUSSION

Figure 3 shows that for a given phantom thickness, the value of the energy imparted per unit exposure-area product, $\omega(z)$, is strongly dependent on photon energy at the lower photon energies. As the photon energy increases, however, the value of $\omega(z)$ generally reaches a plateau value and becomes relatively independent of photon energy. This behavior indicates that for heavily filtered x-ray beams at high kV’s, the energy imparted to the phantom normalized to the entrance skin exposure will be relatively independent of the x-ray spectrum. If the x-ray spectrum contains lower-energy photons, it is the factors that harden an x-ray beam (e.g., small tube anode angles) that will cause $\omega(z)$ to increase, while factors that soften the x-ray beam (e.g., large ripple factors) will reduce $\omega(z)$.

Table I lists the values of $\omega(z)$ for different tube voltages and phantom thicknesses. At a given kV, values of $\omega(z)$ to a water phantom with thickness $z$ cm are given by $\omega(z) = \alpha \times \text{HVL} + \beta$, where the HVL is expressed in mm Al and $\omega(z)$ is in J R$^{-1}$ cm$^{-2}$.

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<th>5 cm</th>
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<th>15 cm</th>
<th>20 cm</th>
<th>25 cm</th>
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water equivalent thickness, $z$, of the patient will permit the appropriate value of $\omega(z)$ to be determined. The product of the free-in-air exposure at the patient entrance surface and the x-ray beam area at this location yields the exposure-area product, EAP. The energy imparted to the patient may then be computed using the expression $e = \omega(z) \times \text{EAP}$. An exposure area meter mounted at the collimator of the x-ray tube may also be used to obtain a direct estimate of the EAP.27–30

Fig. 8. Energy imparted as a function of tube voltage for representative x-ray examinations. The entrance skin exposures, ESE, values (free in air) used for each calculation are listed in each figure.
Any EAP measurements, however, will need to be corrected to take into account the fraction of the x-ray beam area that actually intercepts the patient.

Our method to estimate energy imparted to patients undergoing diagnostic x-ray examinations has several sources of error. Minor errors in computing energy imparted to patients arise from the use of diverging x-ray beams in clinical radiography and the presence of nonuniformities in x-ray beam intensity due to the heel effect. The former is likely to be of negligible importance whereas the latter could easily be accounted for by experimentally obtaining an average entrance skin exposure over the beam area. This latter method is explicitly done when an exposure-area meter is used to quantify the amount of radiation incident on the patient.

Major errors in determining energy imparted to patients result when estimating the equivalent water phantom thickness, , and due to the implicit differences between a (finite) heterogeneous patient and a semi-infinite homogeneous water phantom (Fig. 2). The data presented in Fig. 4 and Fig. 8 indicate that for many clinical applications, the precise value of the water phantom thickness has a small effect on the resultant value of energy imparted. The data presented in Fig. 8 show very good agreement at the lowest kV values, whereas at higher kV’s, the water phantom model generally overestimated the energy imparted values. Given that (lateral) scatter becomes increasingly important at higher kV values, these results suggest that the major source of error may be the use of a water phantom model with infinite lateral extent.

Table II shows a summary of the energy imparted values to patients undergoing a number of radiologic examinations. These values of energy imparted are only approximate indicators of patient risk since they do not take into account the radiosensitivities of the irradiated organs or the age demographics of the irradiated patients. Nevertheless, studies have shown that energy imparted can correlate reasonably well with patient risk for examinations ranging from chest x rays to barium enemas. For uniform whole body exposure in an adult reference man (70 kg), an absorbed energy of 1 J would correspond to an effective dose of 14 mSv. For non-uniform irradiation normally encountered in diagnostic radiology, energy imparted data may be converted into effective dose equivalents by using body region specific / conversion factors, which were used to generate the data in the last column of Table II. Energy imparted to a patient examination range from a few mJ for simple chest examinations to several hundreds of mJ for fluoroscopic examinations lasting ten minutes. The corresponding values of effective dose equivalent range from tens of μSv to tens of mSv.

Energy imparted to patients undergoing diagnostic examinations may be used to study the relationship between the image quality and the patient dose, particularly with the advent of digital radiographic imaging equipment such as photostimulable phosphors and photoconductors. Digital detectors normally permit the radiation exposure to be freely selected by the operator, as opposed to fixed exposures required to generate satisfactory radiographs screen-film combinations. As a result, a simple and convenient method for estimating patient doses permits a systematic investigation of how patient dose and image quality change as radiographic technique factors change. The patient dosimetry method developed in this study is well suited for this purpose since it requires a minimum of effort and no special measurement equipment beyond that normally found in diagnostic radiology departments and has an accuracy that is likely to be satisfactory for most patient dosimetric applications.

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